Towards a UTP semantics for Modelica

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Horizon 2020 Programme

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Future work

INTO-CPS project

- integrated tool-chain for MBD of Cyber-Physical Systems
- “multi-modelling” with heterogeneous models and languages
- SysML for high-level system modelling
- VDM-RT for modelling discrete controllers
- Modelica and 20-sim for continuous system dynamics
- FMI for multi-modelling of heterogeneous systems
- four industrial case studies
  - route simulation in electric cars (TWT)
  - agricultural robot (Agro Intelligence)
  - railways (ClearSy)
  - HVACs in smart buildings (UTRC)
- semantic integration using Unifying Theories of Programming

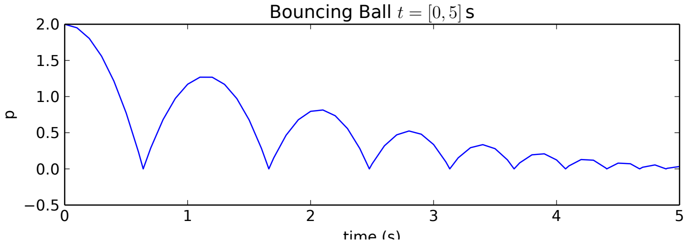
Modelica language

- industrial language for modelling hybrid dynamical systems
- provides language of continuous blocks with connections
- based on hybrid differential-algebraic equations
- combines DAEs with an event handling mechanism
- triggers based on continuous variable conditions
- can cause discontinuities in evolution
- implementations include:
  - Dymola
  - Wolfram SystemModeler
  - MapleSim
  - OpenModelica (INTO-CPS partner)
- incomplete formal semantics

Example 1: Bouncing Ball

Bouncing ball in Modelica

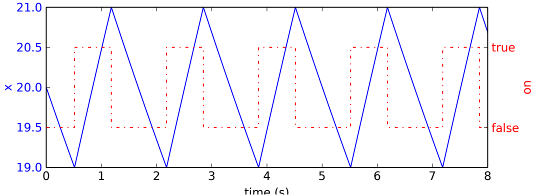
```
model BouncingBall
  Real p(start=2,fixed=true), v(start=0,fixed=true);
equation
  der(v) = -9.81;
  der(p) = v;
  when p <= 0 then
    reinit(v, -0.8*v);
  end when;
end BouncingBall;
```



Example 2: Thermostat

Thermostat in Modelica

```
model Thermostat
  Real x(start=20,fixed=true);
  Boolean on(start=false,fixed=true);
equation
  when x < 19 then
    on = true;
  elsewhen x > 21 then
    on = false;
  end when;
  der(x) = if on then 5 - 0.1*x else -0.1*x;
end Thermostat;
```



UTP in brief

- alphabetised relational calculus – everything is a relation
- expressed as predicates over input, output variables (x / x')
- predicates encode the set of observable behaviours

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x := v &\triangleq x' = v \wedge y' = y \\
P \,;\, Q &\triangleq \exists\, x_0 \bullet P[x_0/x'] \wedge Q[x_0/x] \\
P \lhd b \rhd Q &\triangleq (b \wedge P) \vee (\neg b \wedge Q) \\
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- how to go beyond simple imperative behaviour?
- UTP theories to isolate paradigmatic aspects of a language
- compose theories to produce heterogeneous semantic models
- e.g. object-orientation, concurrency, real-time, ODEs

UTP theories

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   - e.g. $ti, ti' : \mathbb{R}$ to represent start/end time

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   - e.g. $ti' \geq ti$ – time moves forward
   - often expressed as idempotent, monotone functions over preds
   - ensures that healthy predicates form a complete lattice

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3. signature
   - the operators of the language
   - e.g. $\mathsf{Wait}\, n \triangleq ti' = ti + n \wedge v' = v$
   - closed under the healthiness conditions (well-behaved)

Approach

1. create a hybrid relational calculus
   - extends alphabetised relational calculus
   - inspired by Hybrid CSP and Duration Calculus
   - combine continuous variables and discrete i/o variables
   - imperative operators from relational calculus
   - differential equations and pre-emption
   - mechanised in Isabelle/UTP proof assistant

Approach

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   - imperative operators from relational calculus
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   - mechanised in Isabelle/UTP proof assistant
2. define semantic mapping from Modelica to hybrid relations
   - currently a direct semantics for flattened hybrid DAEs
   - describe evolution of ODEs and DAEs
   - elaborate event handling mechanism

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Motivation

- augment relational calculus with continuous variables
- support modelling using differential equations $\langle \dot{x} = \mathcal{F}(x, \dot{x}) \rangle$
- retain standard discrete operators defs – e.g. $P \,;\, Q$, $x := v$
- characterise continuous behaviour with healthiness conditions
- combine with other theories, e.g. CSP and reactive designs
- inspirations:
  - Hybrid CSP (He, Zhan et al.) – DAEs and pre-emption
  - HRML (He) – tri-partite alphabet
  - Duration Calculus (Zhu et al.) – interval operator
  - Timed Reactive Designs (Hayes et al.)

Hybrid relational calculus

- formalise key operators of hybrid behaviour for Modelica
- we begin with a kernel language of imperative hybrid programs
- operators given a semantics in the theory of hybrid relations
- discrete relational operators
  - sequential composition — $P \mathbin{;} Q$
  - assignment — $x := v$
  - if-then-else conditional — $P \lhd b \rhd Q$
  - iteration — P^* and P^ω
- continuous evolution operators
  - DAE — $\langle\, \underline{\dot{v}}_1 = f_1;\ \cdots;\ \underline{\dot{v}}_n = f_n \mid B \,\rangle$
  - pre-emption — $P \,[\, B \,]\, Q$
  - interval (continuous invariant) — $\llbracket P \rrbracket$

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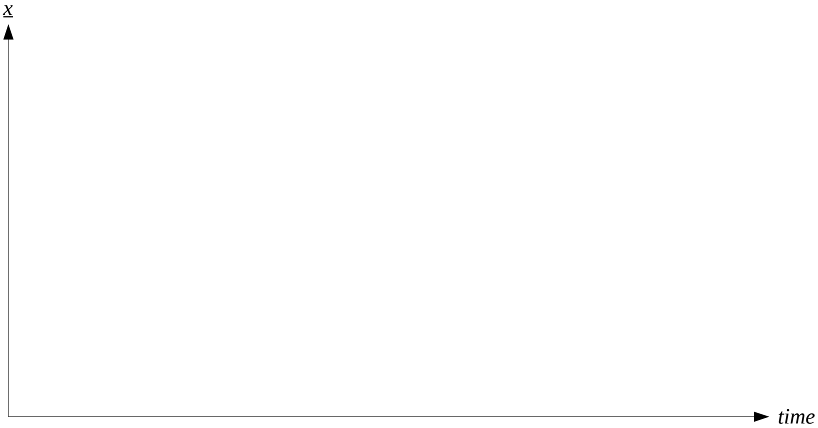
Bouncing ball in hybrid relational calculus

$$p, v := 2, 0 \; ; \left(\left\langle \, \dot{\underline{p}} = \underline{v}; \; \dot{\underline{v}} = -9.81 \, \right\rangle \left[\, \underline{p} \leq 0 \, \right] v := -v * .8 \right)^{\omega}$$

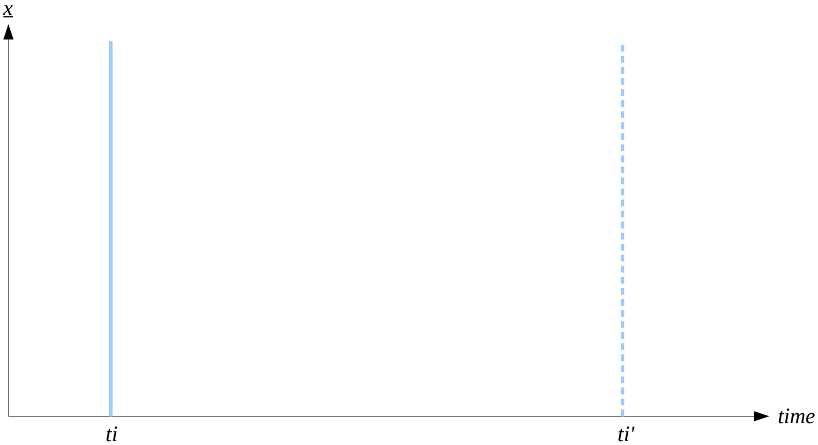
UTP theory of hybrid relations

- model for hybrid relational calculus
- $\alpha(P) = \mathrm{in}\alpha(P) \cup \mathrm{out}\alpha(P) \cup \mathrm{con}\alpha(P)$
- $x, x' : T$ discrete input and output variables
- $\underline{x} : \mathbb{R}_{\geq 0} \rightarrow T$ total continuous variable trajectories
- $ti, ti' : \mathbb{R}_{\geq 0}$ record beginning and end of current observation
- continuous variables range over right-open interval $[ti, ti')$
- accompanied by discrete copies at ends of interval
- continuous linking invariants: $x = \underline{x}(ti)$ and $x' = \lim_{t \to ti'} (\underline{x}(t))$

Healthy continuous observations

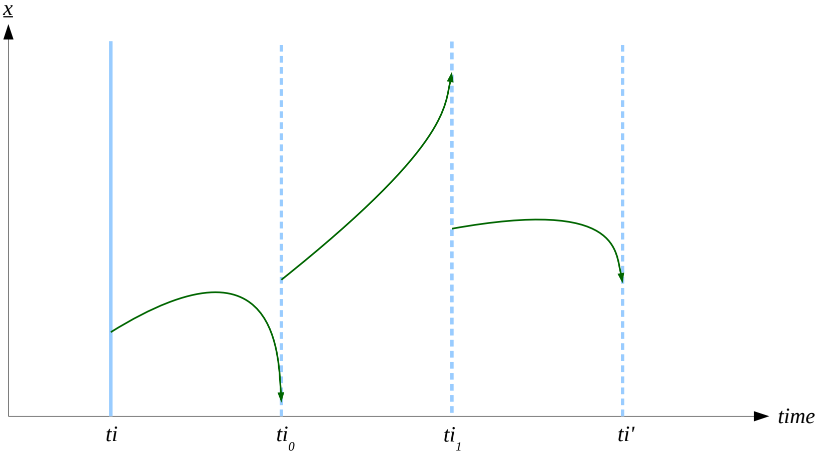


HCT1: Interval has start and end



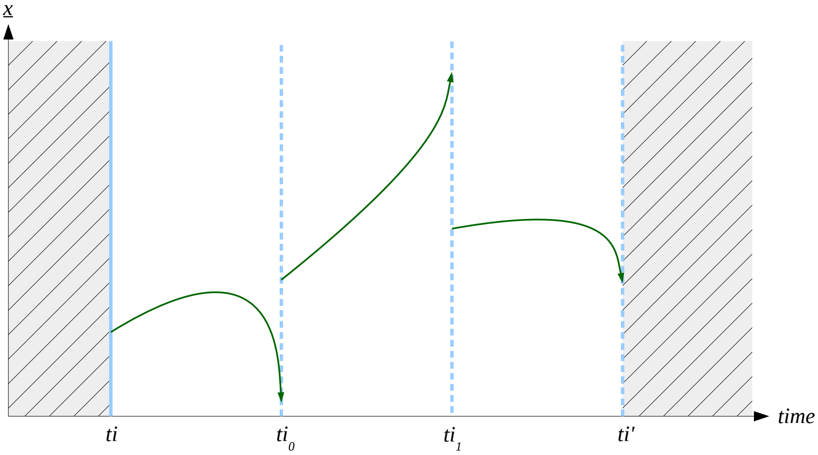
$$\textbf{\textit{HCT1}}(P) \triangleq P \wedge ti \leq ti'$$

HCT2: Trajectories are piecewise continuous



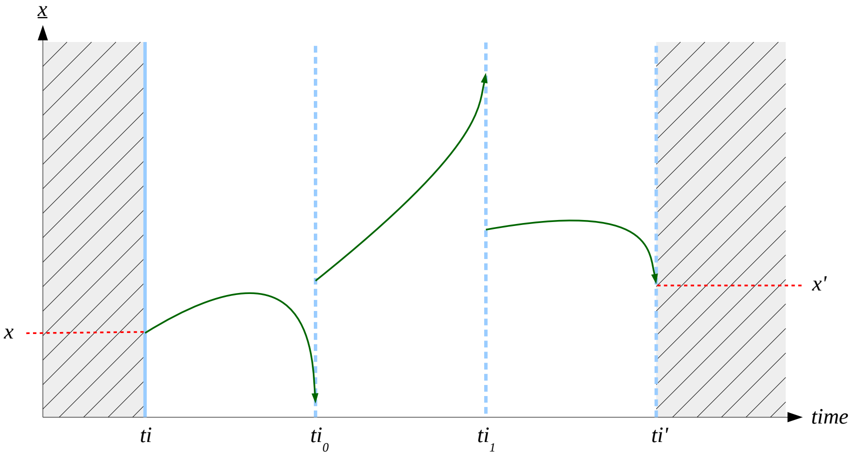
$$\textbf{\textit{HCT2}}(P) \triangleq P \wedge \left(ti < ti' \Rightarrow \left(\begin{array}{l} \exists\, I : \mathbb{R}_{\mathsf{oseq}} \bullet \operatorname{ran}(I) \subseteq \{ti \ldots ti'\} \\ \wedge \{ti, ti'\} \subseteq \operatorname{ran}(I) \wedge \\ \wedge (\forall\, n < \#I - 1 \bullet \underline{x} \ \mathsf{cont\text{-}on}\, [I_n, I_{n+1}]) \end{array} \right) \right)$$

HCT3: Observations are past and future independent



$$\textbf{\textit{HCT3}}(P) \triangleq \bigsqcap t \notin [ti, ti'), v \in \mathsf{type}(\underline{x}) \bullet (P \wedge \underline{x}(t) = v)$$

Continuous linking invariants



$x = \underline{x}(ti)$ and $x' = \lim_{t \to ti'} (\underline{x}(t))$

Healthiness conditions summary

$$
\begin{aligned}
\textbf{\textit{HCT1}}(P) &\triangleq P \land ti \leq ti' \\
\textbf{\textit{HCT2}}(P) &\triangleq P \land \left(ti < ti' \Rightarrow \left(\begin{array}{l} \exists\, I : \mathbb{R}_{\mathsf{oseq}} \bullet \mathrm{ran}(I) \subseteq \{ti \ldots ti'\} \\ \quad \land \{ti, ti'\} \subseteq \mathrm{ran}(I) \land \\ \quad \land (\forall\, n < \#I - 1 \bullet \\ \qquad\qquad \underline{x} \ \textsf{cont-on}\, [I_n, I_{n+1})) \end{array} \right) \right) \\
\textbf{\textit{HCT3}}(P) &\triangleq \bigsqcap t \notin [ti, ti'), v \in \textsf{type}(\underline{x}) \bullet (P \land \underline{x}(t) = v) \\
\textbf{\textit{HCT}}(P) &\triangleq \textbf{\textit{HCT1}} \circ \textbf{\textit{HCT2}} \circ \textbf{\textit{HCT3}}(P)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{R}_{\mathsf{oseq}} &\triangleq \{x : \textsf{seq}\, \mathbb{R} \mid \forall\, n < \#x - 1 \bullet x_n < x_{n+1}\} \\
f \ \textsf{cont-on}\, [m, n) &\triangleq \forall\, t \in [m, n) \bullet \lim_{x \to t} f(x) = f(t)
\end{aligned}
$$

Behaviours

- **instantaneous behaviour**
  - instantaneous observations – $ti' = ti$
  - do not contribute to trajectories ($[ti, ti) = \emptyset$)
  - assign values only to discrete variables
  - described by imperative and concurrent programming operators
  - sequential behaviours $P \,;\, Q$ occupy instant ti
- **continuous time behaviour**
  - non-zero observation duration – $ti' > ti$
  - evolution described by piecewise continuous functions
  - discrete copies of continuous variables at beginning and end
  - described by systems of ODEs and DAEs
  - no sharing of instants between $P \,;\, Q$

Discrete denotational semantics

- standard definitions

$$
\begin{aligned}
x := v &\triangleq x' = v \wedge y' = y \\
P \; ; \; Q &\triangleq \exists\, x_0 \bullet P[x_0/x'] \wedge Q[x_0/x] \\
P \lhd b \rhd Q &\triangleq (b \wedge P) \vee (\neg b \wedge Q) \\
P^{*} &\triangleq \nu X \bullet P \; ; \; X
\end{aligned}
$$

- sequential composition takes the trajectory conjunctions

Continuous denotational semantics

$$\lceil P \rceil \triangleq \textbf{\textit{HCT2}}(\ell > 0 \wedge (\forall \underline{t} \in [ti, ti') \bullet P \,@\, \underline{t}))$$

$$[\![P]\!] \triangleq \lceil P \rceil \wedge \bigwedge_{\underline{v} \in \mathrm{con}\alpha(P)} (v = \underline{v}(ti) \wedge v' = \lim_{t \to ti'} (\underline{v}(t))) \wedge I\!I_{\mathrm{dis}\alpha(P)}$$

$$\langle\, \dot{\underline{v}}_1 = f_1;\ \cdots;\ \dot{\underline{v}}_n = f_n \,|\, B \,\rangle \triangleq$$
$$[\![(\forall i \in 1..n \bullet \dot{\underline{v}}_i(\underline{t}) = f_i(\underline{t}, \underline{v}_1(\underline{t}), \cdots, \underline{v}_n(\underline{t}))) \wedge B]\!]$$

$$P\,[\,B\,]\,Q \triangleq (Q \lhd B \,@\, ti \rhd (P \wedge \lceil \neg B \rceil)) \vee ((\lceil \neg B \rceil \wedge B \,@\, ti' \wedge P)\,;\,Q)$$

- **Theorem:** all operators are ***HCT*-closed**

Mechanisation

- based on Isabelle/UTP and the Multivariate Analysis package
- mimics syntax contained in paper
- proof support for alphabetised and hybrid relational calculi
- real numbers based on Cauchy sequences
- support for limits, ODEs, and their solutions
- proved key properties of HCT and signature

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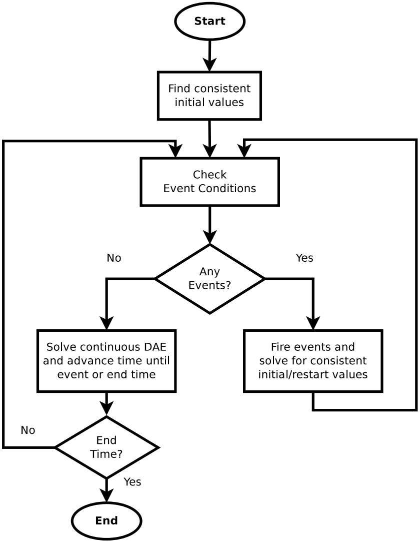
Event iteration cycle



Denotational semantics of Modelica

- currently defined as a mapping on flat Modelica

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- variables: dynamic (x), algebraic (y), discrete (q)
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- $k \in \mathbb{N}_{>0}$ conditional DAEs:
  - differential equations $\dot{x} = \mathcal{F}_i(x, y, q)$ for $i \in 1..k$
  - algebraic equations $y = \mathcal{B}_i(x, y, q)$ for $i \in 1..k$
  - boolean DAE guards $\mathcal{G}_i(x, y, q)$ for $i \in 1..k-1$

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  - boolean DAE guards $\mathcal{G}_i(x, y, q)$ for $i \in 1..k-1$
- $l \in \mathbb{N}$ boolean event conditions $\mathcal{C}_i(x, y, q)$ for $i \in 1..l$
- $m \in \mathbb{N}$ conditional discrete equation blocks
  - discrete-event guards $\mathcal{H}_{i,j}(x, y, q, q_{pre})$ for $i \in 1..m$, $j \in 1..n$
  - discrete algorithms $\mathcal{P}_{i,j}(x, y, q, q_{pre})$ for $i \in 1..m$, $j \in 1..n$

High-level semantic mapping

$$
\begin{aligned}
\mathcal{M} &= \mathsf{Init}\ ;\ (\mathsf{DAE}\,[\,\mathsf{Events}\,]\,\mathsf{Discr})^{\omega} \\
\mathsf{Init} &= \underline{x}, \underline{y}, q := u, v, w \\
\mathsf{DAE} &= \left\langle\, \underline{x} = \mathcal{F}_1(\underline{\dot{x}}, \underline{y}, q) \,\middle|\, \mathcal{B}_1(\underline{x}, \underline{y}, q) \,\right\rangle \lhd \mathcal{G}_1 \rhd \cdots \\
& \quad \lhd \mathcal{G}_{n-1} \rhd \left\langle\, \underline{\dot{x}} = \mathcal{F}_n(\underline{x}, \underline{y}, q) \,\middle|\, \mathcal{B}_n(\underline{x}, \underline{y}, q) \,\right\rangle \\
\mathsf{Events} &= \bigvee_{i \in \{1..k\}} \mathcal{C}_i(\underline{x}, \underline{y}, q) \neq \mathcal{C}_i(x, y, q) \\
\mathsf{Discr} &= \textbf{\textit{var}}\ q_{pre} \bullet \\
& \quad \textbf{\textit{until}}\ q_{pre} = q\ \textbf{\textit{do}} \\
& \qquad q_{pre} := q\ ; \\
& \qquad \mathcal{P}_{1,1}(\underline{x}, \underline{y}, q, q_{pre}) \lhd \mathcal{H}_{1,1}(\underline{x}, \underline{y}, q, q_{pre}) \rhd \mathcal{P}_{1,2}(\underline{x}, \underline{y}, q) \lhd \cdots\ ;\ \cdots\ ; \\
& \qquad \mathcal{P}_{m,1}(\underline{x}, \underline{y}, q, q_{pre}) \lhd \mathcal{H}_{m,1}(\underline{x}, \underline{y}, q, q_{pre}) \rhd \mathcal{P}_{m,2}(\underline{x}, \underline{y}, q, q_{pre}) \lhd \cdots\ ; \\
& \quad \textbf{\textit{od}}
\end{aligned}
$$

Semantics of bouncing ball

Example

Bouncing ball semantics in hybrid relational calculus

$$
\begin{array}{l}
h, v, c := 2, 0, \mathit{false} \,; \\
\quad (\left\langle\, \underline{\dot{v}} = -9.81;\; \underline{\dot{h}} = \underline{v} \,\right\rangle \\
\qquad [(\underline{h} < 0) \neq (h < 0)] \\
\qquad \textbf{\textit{var}}\ c_{pre} \bullet \\
\qquad\quad \textbf{\textit{until}}\ (c_{pre} = c)\ \textbf{\textit{do}} \\
\qquad\qquad c_{pre} := c \,;\, c := h < 0 \,; \\
\qquad\qquad v := -0.8 \cdot v \lhd c \land \neg c_{pre} \rhd I\!I \\
\qquad\quad \textbf{\textit{od}})^{\omega}
\end{array}
$$

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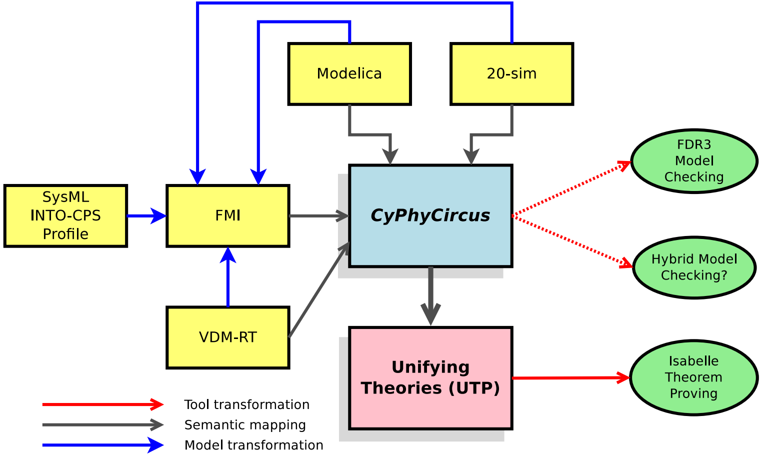
- completion of the Modelica semantic mapping
  - e.g. full expression AST
- compositional mapping for blocks and wires
- testing semantics on more substantive examples
- theorem proving support for Modelica
  - are my initial value constraints consistent?
  - does the event iteration cycle terminate?
  - does this algorithm satisfy an invariant?
- enrich hybrid relations $\longrightarrow$ hybrid reactive designs
- use the latter to build a lingua franca – CyPhyCircus
- enable integration of discrete controllers with continuous plant

INTO-CPS semantic integration



CyPhyCircus

- minimal CSP extension with hybrid behaviour
- semantics based on hybrid reactive designs $\boldsymbol{HR}(P \vdash Q)$
- enable modelling of reactive and concurrent hybrid systems
- give a model to Hybrid Hoare Logic using interval operator(?)

$$
\begin{aligned}
P, Q ::= \; & \mathsf{Skip} \mid \mathsf{Stop} \mid P \,;\, Q \mid P \lhd b \rhd Q \mid P \sqcap Q \mid x := e \\
& \mid P^{*} \mid P^{\omega} \mid \langle\, \underline{\dot{x}} = f(x, \underline{x}, \underline{\dot{x}}) \mid b(x, \underline{x}) \,\rangle \mid P \,[\, b(x, \underline{x}) \,]\, Q \\
& \mid \mathbin{[\!]}_{i \in I} \; a_i?x \rightarrow P(x) \mid a!e \rightarrow P \mid P \mathbin{\triangle} Q \mid P \parallel Q
\end{aligned}
$$

Links

- Isabelle/UTP git repository: https://github.com/isabelle-utp/utp-main/tree/shallow.2016
- INTO-CPS project: http://into-cps.au.dk/

Thanks for listening!

Horizon 2020 Programme