Probabilities and Mixed-Criticalities: the Probabilistic C-Space

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Why considering a probabilistic analysis ?

1. Probabilistic C-Space schedulability analysis
2. Mixed-criticality with probabilities: define the probability of being in a given criticality level
3. Probabilistic Sensitivity Analysis

In this presentation we explore Item (2) with the help of the probabilistic C-Space

Motivation

Predicting Task Execution Time is difficult:

- execution variability (inputs, environment, tasks)
- additional (and unpredictable) delays in task executions from the system
- interferences from system elements (included other tasks)

pWCET to model task execution variability is an alternative to Worst-Case Execution Times (WCETs)

Contributions

Provide probabilistic guarantees for MC

- **formalize** scheduling problem with criticalities through probabilities
- **pC-Space**: pWCETs applied to construct probabilistic version of C-space
- provide **initial evaluation** flexibility of probabilistic models and probabilistic scheduling for the mixed-criticality problem

Outline

1 Introduction

2 Modeling

3 Schedulability and C-space with probabilities

Computational Model

Task model and Scheduling

$\tau_i = (\mathcal{C}_i, T_i, D_i)$
Set of n periodic tasks $\Gamma = \{\tau_1, \ldots, \tau_n\}$
$H = \text{lcm}(T_1, \ldots, T_n)$
Γ is scheduled with EDF

Probabilistic Worst-Case Execution Time

The $\text{pdf}_{\mathcal{C}_i}$ is the probability distribution function (pdf) representation of the random variable $\mathcal{C}_i$.

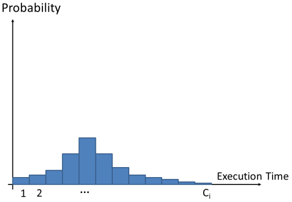
$$\text{pdf}_{\mathcal{C}_i} = \begin{pmatrix} c_{i,1} & \dots & c_{i,k_i} \\ P(\mathcal{C}_i = c_{i,1}) & \dots & P(\mathcal{C}_i = c_{i,k_i}) \end{pmatrix}$$

- $\text{pdf}_{\mathcal{C}_i}(c_{i,r}) = P(\mathcal{C}_i = c_{i,r})$, $\sum_{r=1}^{k_i} \text{pdf}_{\mathcal{C}_i}(c_{i,r}) = 1$; $P(\mathcal{C}_i \leq C_i) = 1$

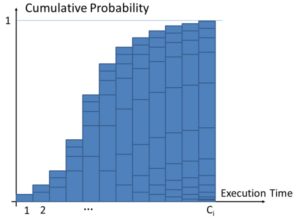
Probabilistic Worst-Case Execution Time

Cumulative distribution function (cdf):

$\mathrm{cdf}_{\mathcal{C}_i}(c) = P(\mathcal{C}_i \leq c) = \sum_{x=1}^{c} \mathrm{pdf}_{\mathcal{C}_i}(x)$



(a) Worst-case execution time histogram representation



(b) Worst-case execution time cdfs

Probabilistic Worst-Case Execution Time

WCET Thresholds

$\mathcal{C}_i \quad \rightarrow$ *WCET thresholds*: $\langle c_{i,k}, p_{i,k} \rangle$, $1 \leq k \leq k_i$

The worst-case value $c_{i,k}$ associated to $p_{i,k}$

$p_{i,k} \stackrel{def}{=} \text{cdf}_{\mathcal{C}_i}(c_{i,k})$: **accuracy/confidence** of WCET $c_{i,k}$

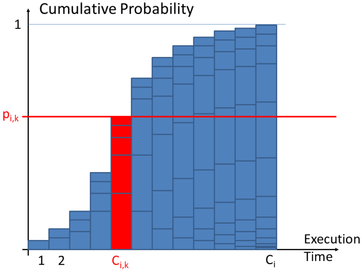


Figure: WCET threshold

Probabilistic Mixed-Criticality

Classical Mixed-critical WCET $C_i = (LO - WCET, HI - WCET)$
LO-WCET by a less pessimistic timing analysis tools, larger HI-WCET by more conservative timing analysis tools

MC analysis

- Most pessimistic assumption: If one HI-criticality task executes beyond its LO-WCET the system considers that all HI-criticality will do the same
- Our approach: define for each task the level of confidence on its Execution Time w.r.t a criticality level

The probability of meeting deadlines is function of the criticality level that depends on the level of confidence on the WCETs

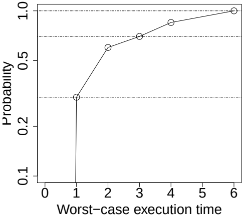
Probability Threshold

- Design parameter β (**probability threshold**) defining the level of confidence for a WCET imposed to a task at a criticality level *crit*
- From β we get $c_i(\beta)$ the corresponding WCET in the cdf of $\mathcal{C}_i$
- $\beta \times 100\%$ of the worst-case execution time experienced by τ_i are below $c_i(\beta)$

Now for any level of confidence β what is the probability to meet deadlines at criticality level *crit* ?

Probability Threshold

We can define a level of confidence for each criticality level and for each task



(a) τ_1

(b) τ_2

(c) τ_3

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Probabilistic Demand Bound Function

$\mathrm{dbf}_{i,j}(t)$: the demand bound function in the interval $[0, t]$:

$$\mathrm{dbf}_{i,j}(t) \stackrel{def}{=} \lfloor \frac{t - D_i}{T_i} + 1 \rfloor \times c_{i,j}$$

- $\mathrm{dbf}_{i,k}$ from $c_{i,k} \leq c_{i,j}$; $\langle \mathrm{dbf}_{i,j}(t), p_{i,j} \rangle$ with $p_{i,j}$ the probability that $\mathrm{dbf}_{i,j}(t)$ upper-bounds τ_i resource demand in $[0, t]$ (not function of t, obtained from the cumulative distribution of $\mathcal{C}_i$)
- All $\langle \mathrm{dbf}_i(t, c_{i,j}), p_{i,j} \rangle$ form a distribution of demand bound functions
- $\mathcal{DBF}_i(t) = (\mathrm{dbf}_i(t, \cdot), p_i(\cdot))$: probabilistic demand bound function (probabilistic demand curve) of τ_i in $[0, t]$

Probabilistic Demand Bound Function

From each $\mathcal{DBF}_i(t)$, we can then compute the application level probabilistic distribution $\mathcal{DBF} = (\text{dbf}(t, \cdot), p(\cdot))$:

$$\mathcal{DBF}(t) = \otimes_i \mathcal{DBF}_i(t),$$

The EDF schedulability condition states:

$$\forall t \in D, \quad \text{dbf}(t) \leq t,$$

with D the set of deadlines within the hyperperiod (this number can be significantly with a linear programming approach (George & Hermant 2009 for constrained deadlines)

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Probabilistic Demand Bound Function

Now what is the probability to satisfy at time t, $dbf(t) \leq t$?

- This can be computed by taking in $\mathcal{DBF}(t)$ the maximum value less than or equal to t to find the associated probability $P(t)$

Hence for all times t_i to consider in $[0, H]$, the probability P to meet meet deadlines with EDF is:

$$P = P(t_1) \times P(t_2) \times \ldots$$

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Probability to meet deadlines at level of confidence β

- A criticality level is defined with it associated level of confidence β
- From β, we constrain execution times of each task to a maximum value
- We therefore update $\mathcal{DBF}_i(t)$ to take into account only possible execution times

$$P_\beta = P_\beta(t_1) \times P_\beta(t_2) \times \dots$$

Probabilistic Schedulability

Theorem (β-EDF Schedulability)

Any probabilistic task set $\{\tau_1, \tau_2, \ldots, \tau_n\}$ *with in a dbf distribution* $\mathcal{DBF}(t_i)$ *at time* t_i *is schedulable with a probability* P_β *under EDF with a level of confidence* $\beta \in]0, 1]$*, with* S *the set of significant deadlines in* $[O, H]$*and* :

$$P_\beta = P_\beta(t_1) \times P_\beta(t_2) \times \ldots$$

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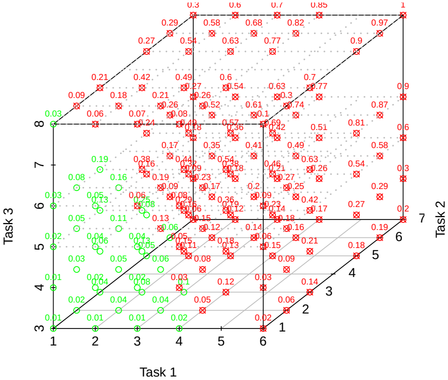
Probabilistic C-Space [1/2]

- We can extend the β-EDF Schedulability condition to characterize the pC-space
- A vector of execution times $\overline{c} = \{c_1, c_2, \ldots\}$ in the pc-Space can be associated to a corresponding probability in all distributions $\mathcal{DBF}(t_i)$, $t_i \in S$



Probabilistic C-Space [2/2]

The probabilities within the pC-Space can be interpreted in various ways:

1. as the confidence of not passing the WCET thresholds of $\overline{c}$;
   - $p(crit) = p_1(crit) \times p_2(crit) \times \dots$
   - $1 - p(crit)$ the possibility of changing that level
2. the confidence on system schedulability P: points inside the region are schedulable, confidence of at least P
3. the confidence β on task worst-case behavior; β is the probability of passing $\overline{c}(\beta)$

Conclusion and Future Work

Combining probabilities and mixed-criticality problem through:

- probabilistic C-space
- probabilistic sensitivity analysis

Formalization and initial ideas for helping designing and applying MC scheduling policies

Conclusion and Future Work

In the future:

- define probabilistic sensitivity analysis in terms of change strategies and effect evaluation
- leverage the information from the probabilistic models (pWCET distributions and confidences)
- provide system design feedbacks for an optimal (at least suboptimal) system resource utilization for different criticalities

Thank you!

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