Fixed Priority until Zero Laxity (FPZL) Schedulability Analysis

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Research scope

- **Homogeneous Multiprocessor Real-Time Systems**
- **Global scheduling**
  - Single global run-queue
  - Pre-emption and migration
- **Based on fixed task-priority scheduling**
  - All jobs of a task have the same fixed priority
- **Add minimally dynamic priorities**
  - Promote the priority of any job that would otherwise inevitably miss its deadline (zero-laxity)

Motivation

- **Improve upon the effectiveness of global FP scheduling**
  - Dynamic priority algorithms
    - Potentially much more effective than fixed task-priority algorithms in terms of the tasksets that can be scheduled
    - But can have significantly larger overheads e.g. theoretically optimal algorithms with n -1 context switches per job release
- **Avoid significant increase in complexity or number of context switches**
  - FPZL: Zero-Laxity rule applied to global FP scheduling
    - When remaining execution time equals time to deadline, task must run or the deadline will be missed - so priority promoted
    - At most **one** change in priority per job release
    - At most **two** pre-emptions per job release

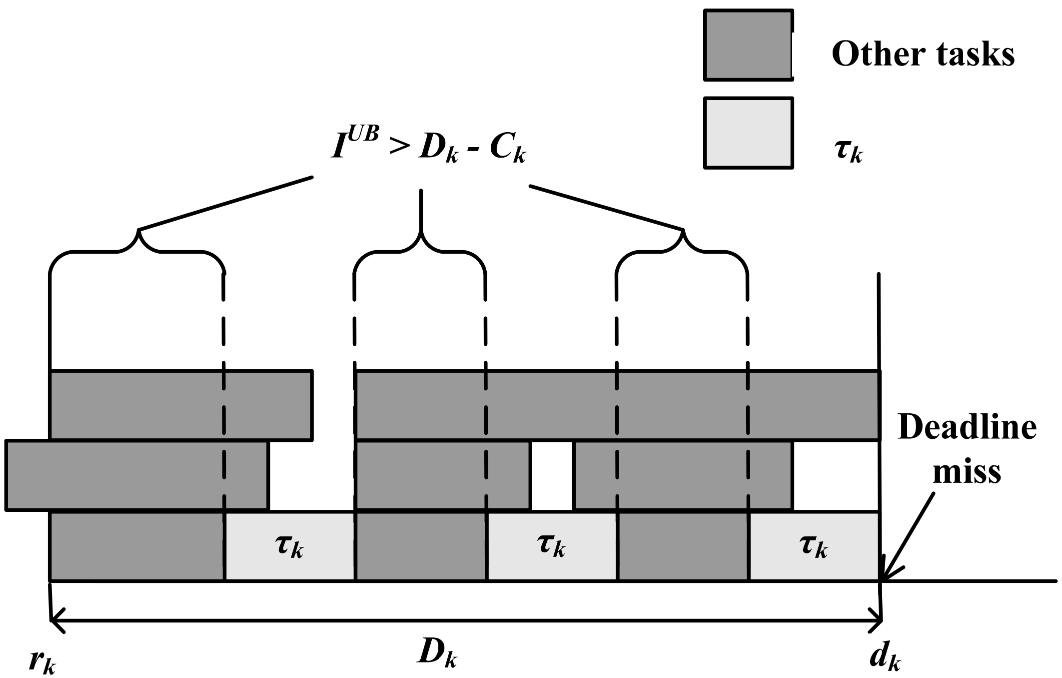
Outline

- **System model, terminology, and definitions**
- **Recap on schedulability tests for global FP scheduling**
- **Schedulability tests for FPZL**
- **Improving the tests by bounding execution time in the zero-laxity state**
- **Empirical results**
  - Schedulability test performance
  - Algorithm performance (simulation)
- **Comparison with previous work on RMZL**
- **Summary and conclusions**

System model

- **Multiprocessor system**
  - m identical processors
  - FPZL scheduling (global FP pre-emptive scheduling + priority promotion at zero-laxity)
  - Migration is permitted, but a job can only execute on one processor at a time
- **Sporadic task model**
  - Static set of n tasks τ_i with priorities $1..n$
  - Bounded worst-case execution time C_i
  - Sporadic/periodic arrivals: minimum inter-arrival time T_i
  - Relative deadline D_i (Constrained deadlines $\leq T_i$)
  - Independent

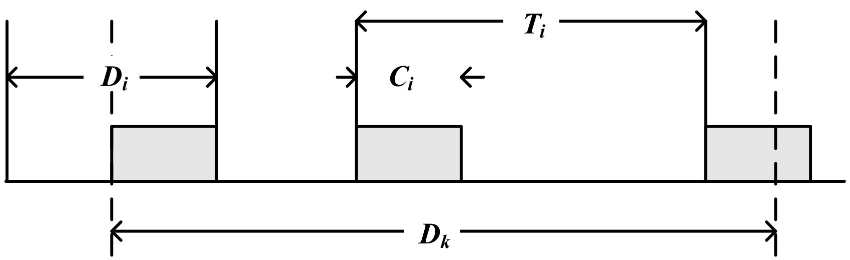
Global FP: Sufficient schedulability tests



- **Fundamental approach (Baker [2])**
  - Problem window in which deadline is missed (e.g. D_k)
  - Necessary condition for deadline miss:
    m processors all occupied for more than D_k - C_k
  - Derive upper bound on interference I^{UB} from other tasks
  - Negate the un-schedulability condition to form a sufficient schedulability test for task τ_k

Deadline analysis for global FP

- **Worst-case scenario for task τ_k (Davis & Burns [16], Guan et al. [20])**
  - At most (m -1) higher priority tasks contribute *carry-in* interference

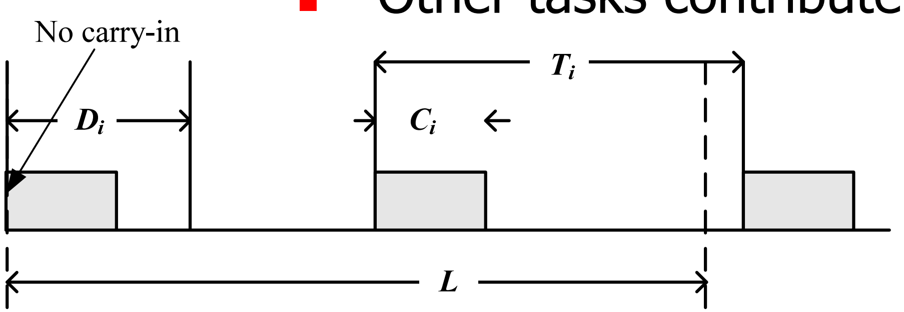


$$I_i^D(L, C_k) = \min(W_i^D(L), L - C_k + 1)$$

$$W_i^D(L) = N_i^D(L)C_i + \min(C_i, L + D_i - C_i - N_i^D(L)T_i)$$

$$N_i^D(L) = \lfloor (L + D_i - C_i)/T_i \rfloor$$

  - Other tasks contribute no carry-in interference



$$I_i^{NC}(L, C_k) = \min(W_i^{NC}(L), L - C_k + 1)$$

$$W_i^{NC}(L) = N_i^{NC}(L)C_i + \min(C_i, L - N_i^{NC}(L)T_i)$$

$$N_i^{NC}(L) = \lfloor L/T_i \rfloor$$

Deadline analysis for global FP

- **Polynomial time test: Deadline Analysis ("DA-LC test") (Davis & Burns [16] based on Bertogna et al. [9], Guan et al [20])**
  - Difference between carry-in and no carry-in interference

$$I_i^{DIFF-D}(L, C_k) = I_i^{D}(L, C_k) - I_i^{NC}(L, C_k)$$

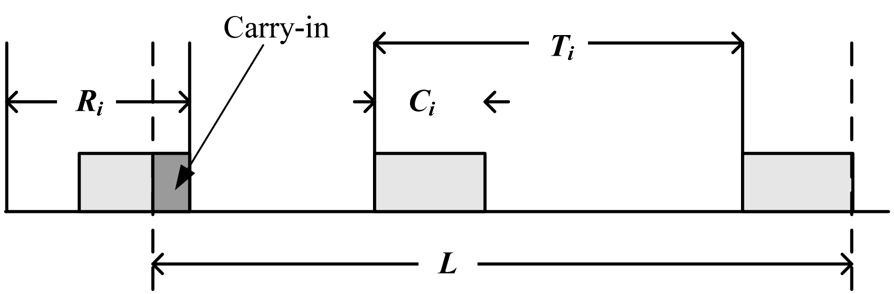
  - Include extra interference from (m – 1) tasks with largest difference between carry-in and no carry-in interference

$$D_k \geq C_k + \left\lfloor \frac{1}{m} \left(\sum_{\forall i \in hp(k)} I_i^{NC}(D_k, C_k) + \sum_{i \in MD(k, m-1)} I_i^{DIFF-D}(D_k, C_k) \right) \right\rfloor$$

  - Schedulability test for each task τ_k

Response Time analysis for global FP

- **Worst-case scenario for task τ_k (Guan et al. [20])**
  - At most (m -1) tasks contribute *carry-in* interference

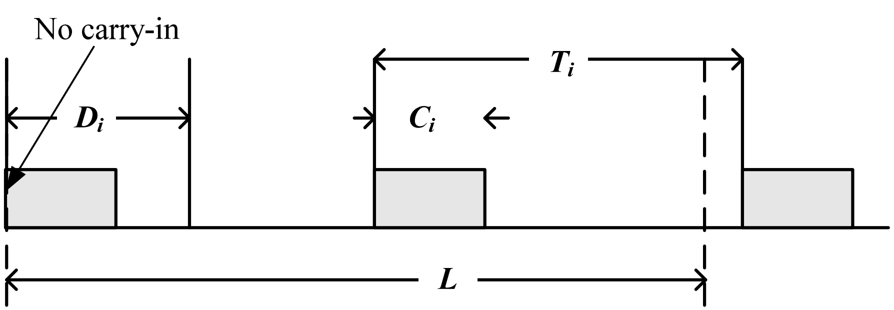


$$I_i^R(L, C_k) = \min(W_i^R(L), L - C_k + 1)$$

$$W_i^R(L) = N_i^R(L)C_i + \min(C_i, L + R_i^{UB} - C_i - N_i^R(L)T_i)$$

$$N_i^R(L) = \lfloor (L + R_i^{UB} - C_i) / T_i \rfloor$$

  - Others contribute no carry-in interference (as before)



$$I_i^{NC}(L, C_k) = \min(W_i^{NC}(L), L - C_k + 1)$$

$$W_i^{NC}(L) = N_i^{NC}(L)C_i + \min(C_i, L - N_i^{NC}(L)T_i)$$

$$N_i^{NC}(L) = \lfloor L / T_i \rfloor$$

Response Time analysis for global FP

- **Pseudo-polynomial time test: Response Time Analysis ("RTA-LC test") (Guan et al [20], based on Bertogna & Cirinei [8])**
  - Difference between carry-in and no carry-in interference

$$I_i^{DIFF-R}(L, C_k) = I_i^{R}(L, C_k) - I_i^{NC}(L, C_k)$$

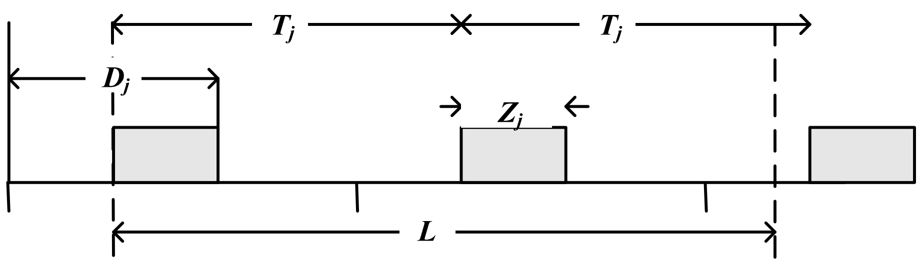
  - Include extra interference from (m – 1) tasks with largest difference between carry-in and no carry-in interference

$$R_k^{UB} \leftarrow C_k + \left\lfloor \frac{1}{m} \left(\sum_{\forall i \in hp(k)} I_i^{NC}(R_k^{UB}, C_k) + \sum_{i \in MR(k, m-1)} I_i^{DIFF-R}(R_k^{UB}, C_k) \right) \right\rfloor$$

Recall dependency on response time upper bounds of higher priority tasks – need to evaluate schedulability in priority order – highest priority first

FPZL Schedulability analysis

- **Differences w.r.t. analysis for global FP**
  - Up to m tasks may be deemed unschedulable but still meet their deadlines due to the zero-laxity rule
  - Tasks executing in the zero-laxity state have an impact on the schedulability of other tasks (assume $Z_j^{UB} = C_j$)



$$I_j^Z(L, C_k) = \min(W_j^Z(L), L - C_k + 1)$$

$$W_j^Z(L) = N_j^Z(L) Z_j^{UB} + \min(Z_j^{UB}, L - N_j^Z(L) T_j)$$

$$N_j^Z(L) = \lfloor L / T_j \rfloor$$

  - Zero-laxity execution immediately proceeds the deadline
    - Equations similar to "no carry-in" case
    - Need only consider lower priority zero-laxity tasks (no increase in interference from higher priority zero-laxity tasks – already of higher priority)

FPZL Schedulability Analysis

- **Deadline Analysis for FPZL (DA-LC test)**

$$D_k \geq C_k + \left\lfloor \frac{1}{m} \left(\begin{array}{l} \sum_{\forall i \in hp(k)} I_i^{NC}(D_k, C_k) + \\ \sum_{i \in MD(k,m-1)} I_i^{DIFF-D}(D_k, C_k) + \\ \sum_{\forall j \in lpzl(k)} I_j^{Z}(D_k, C_k) \end{array} \right) \right\rfloor$$

  - If inequality holds, task is schedulable without priority promotion, otherwise it is a zero-laxity task
  - At most m zero-laxity tasks in a schedulable system
  - Dominates equivalent test for global FP
  - Schedulability needs to be checked lowest priority first to identify which tasks are zero-laxity tasks
  - Polynomial time $O(n^2)$ test of taskset schedulability

FPZL Schedulability Analysis

- **Response Time Analysis for FPZL (RTA-LC test)**

$$R_k^{UB} \leftarrow C_k + \left\lfloor \frac{1}{m} \left(\begin{array}{l} \sum_{\forall i \in hp(k)} I_i^{NC}(R_k^{UB}, C_k) + \\ \sum_{i \in MR(k,m-1)} I_i^{DIFF-R}(R_k^{UB}, C_k) + \\ \sum_{\forall j \in lpzl(k)} I_j^{Z}(R_k^{UB}, C_k) \end{array} \right) \right\rfloor$$

- As before:
  - If $R_k^{UB} \le D_k$, task is schedulable without priority promotion, otherwise it is a zero-laxity task
  - At most m zero-laxity tasks in a schedulable system
  - Dominates equivalent test for global FP
- Problem:
  - Response time upper bound depends on response times of higher priority tasks and the zero-laxity status of lower priority tasks

FPZL Schedulability Analysis

- RTA Solution
  - Response time (and hence zero-laxity status) is monotonically non-decreasing in the response times of higher priority tasks and the zero-laxity status / zero-laxity execution times of lower priority tasks
  - Whenever a zero-laxity task is found – must repeat response time calculations

```
countZL = 0
Initialize all R_k^UB = C_k and Z_k^UB = 0
repeat = true
while (repeat) {
    repeat = false
    for (each priority level k, highest first) {
        Determine R_k^UB according to (18)
        if ( R_k^UB > D_k ) {
            R_k^UB = D_k
            Compute Z_k^UB
            if ( τ_k not marked as a ZL task) {
                mark τ_k as a ZL task
                repeat = true
                countZL = countZL + 1
                if(countZL > m) {
                    repeat = false
                    break (exit for loop)
                }
            }
        }
        [if ( R_k^UB or Z_k^UB differ from prev. values)
            repeat = true]
    }

if (countZL > m)
    return unschedulable
else
    return schedulable
```

Bounding zero-laxity execution time

- **DC-Sustainability**
  - A schedulability test is *DC-Sustainable* provided that
    - Any task that is **schedulable** according to the test with parameters (D,C) remains schedulable when D and C are reduced by the same amount x to $(D\text{-}x,\ C\text{-}x)$
    - Any task that is **unschedulable** according to the test with parameters (D,C) remains unschedulable when D and C are increased by the same amount to $(D+x,\ C+x)$
  - Both FPZL schedulability tests (DA-LC and RTA-LC) are DC-Sustainable
    - Proofs in the paper

Bounding zero-laxity execution time

- **Execution time in the zero-laxity state**
  - DC-Sustainability of the schedulability tests means
    - For each zero-laxity task, we can use a binary search to find the min value of x such that the task is schedulable with parameters (D-x, C-x) without priority promotion
    - x is then an upper bound on the execution time in the zero-laxity state
  - Response Time Analysis
    - Iterative calculation - also need to re-start calculations whenever the response times or execution times in the zero-laxity state change

Empirical Investigation

- **Taskset parameters**
  - Task utilisations generated via UUnifast-Discard
  - Task periods chosen from a log-uniform distribution with a range from min to max period of 1000 (e.g. 1ms to 1 sec)
  - Execution times set from task utilisation and period values
  - Task deadlines chosen from a uniform distribution between execution time and period
  - Total utilisation varied from 0.025*m* to 0.975*m* in steps of 0.025*m*
  - 1000 tasksets generated for each total utilisation level
  - Graphs plot the percentage of tasksets that are schedulable according to each schedulability test against total utilisation

Empirical Investigation

- **Sufficient schedulability tests**
  - Global FP: (DA-LC test and DMPO)
  - Global FP: (DA-LC test and OPA)
  - Global EDF: (EDF-RTA test)
  - EDZL: (EDZL-I test)
  - FPZL: (DA-LC test and OPA)
- **LOAD* necessary infeasibility test**
- **Simulations**
  - Global FP (DMPO, DCMPO)
  - FPZL (DCMPO)
  - EDF
  - EDZL

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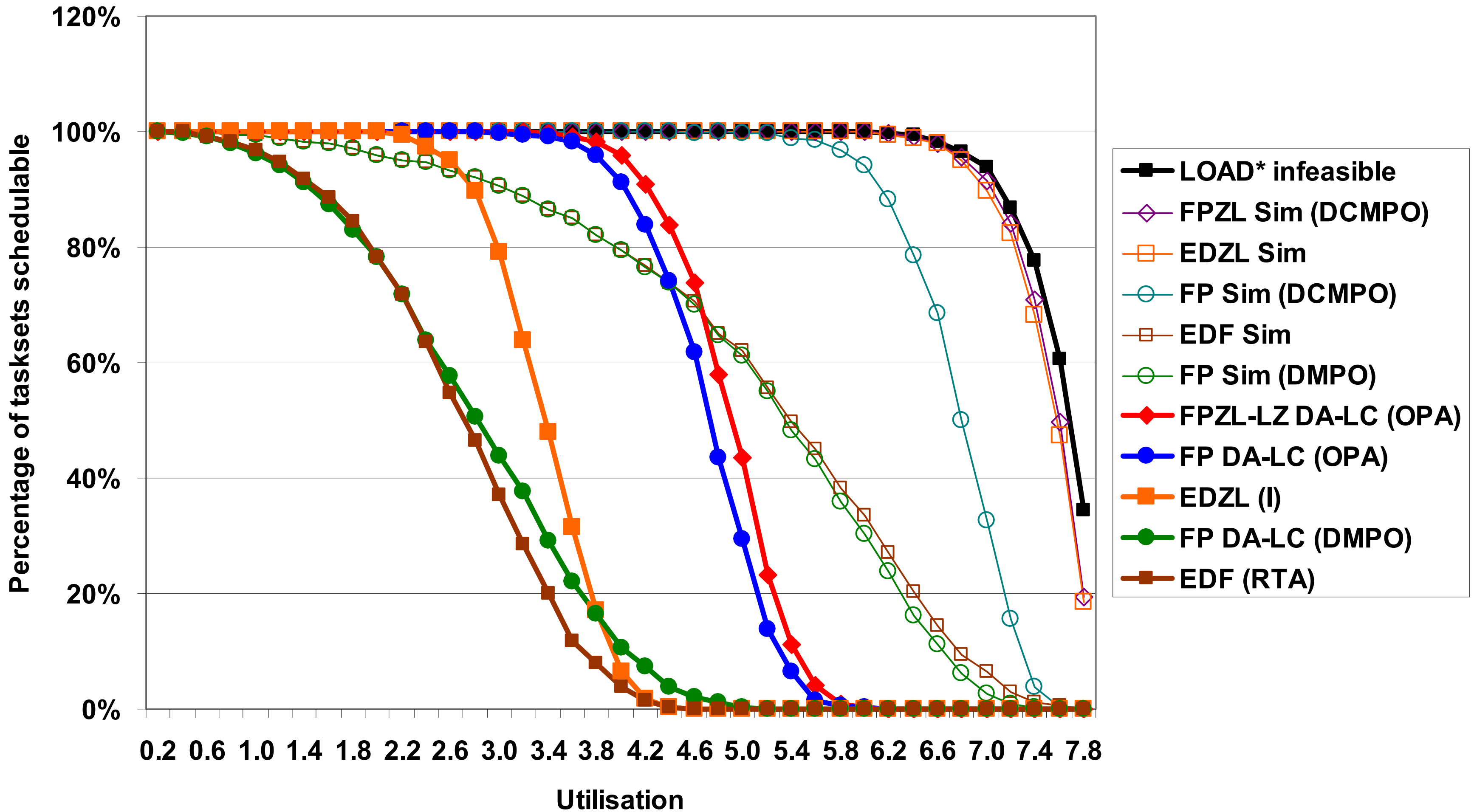
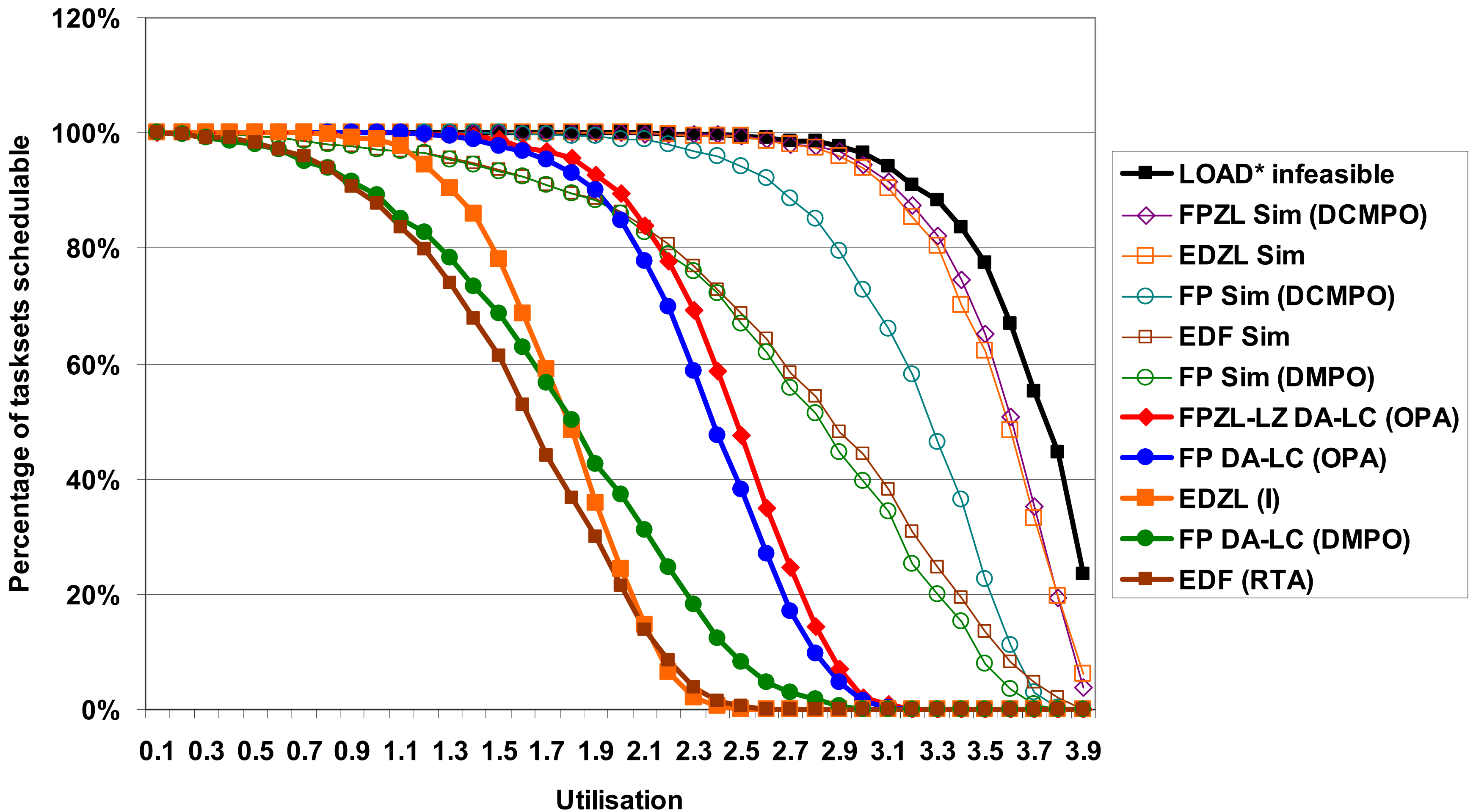
Empirical results:

8 Processors
40 tasks D≤T



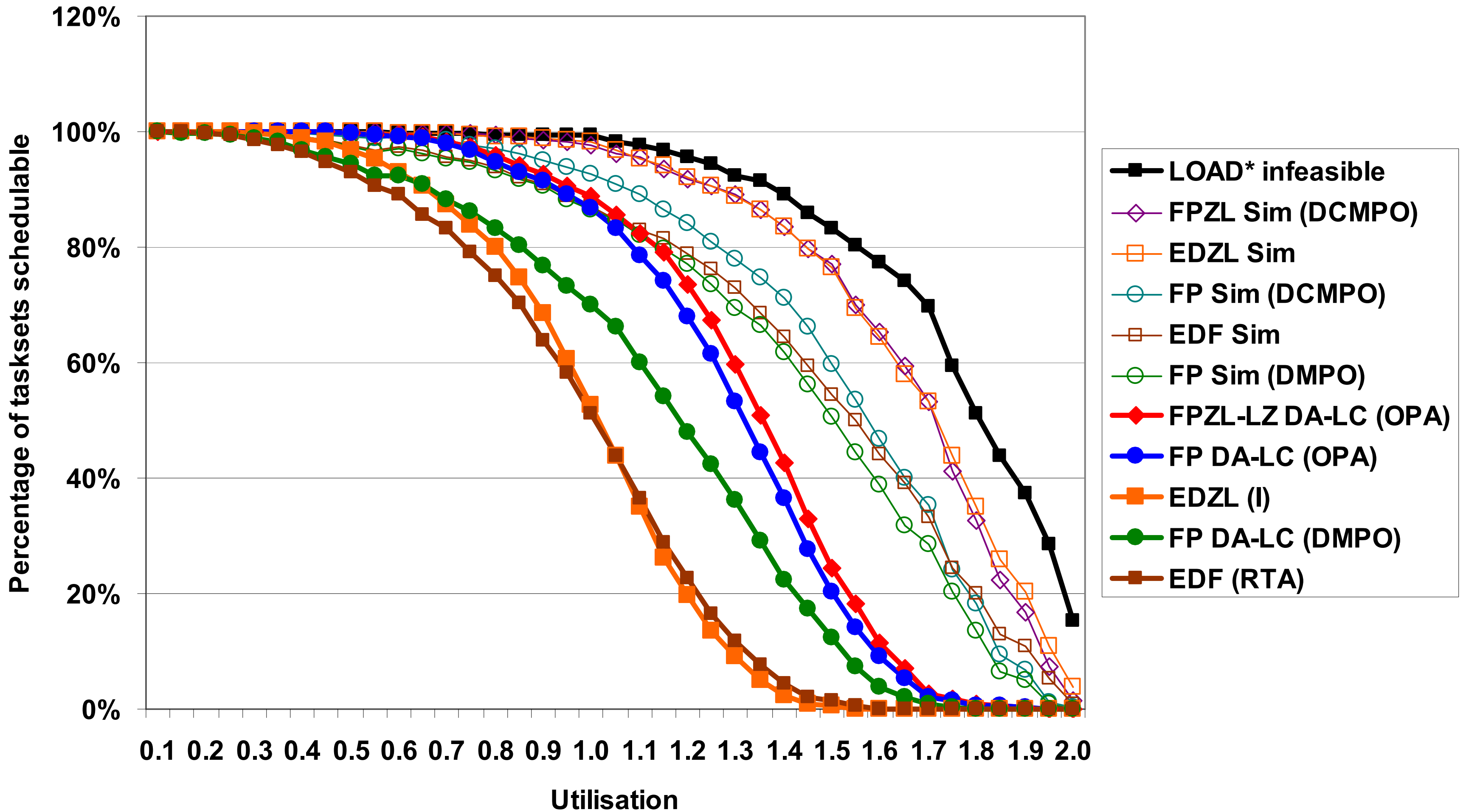
Empirical results:

4 Processors
20 tasks D≤T



Empirical results:

2 Processors
10 tasks D≤T



RMZL and FPZL

- **Related research on RMZL**
  - Originally published in Japanese by Shinpei Kato
  - Now available as a technical report in English
  - RMZL is the same zero-laxity rule applied to global FP scheduling for the “Rate Monotonic” case ($D=T$)
    - Algorithm is the same as FPZL
    - Analysis is simpler but only applicable to the implicit deadline case with RM priority order
    - RMZL analysis assumes every lower priority task can be a zero-laxity task
    - Unfortunately this leads to declining schedulability test performance with an increasing number of tasks
  - FPZL schedulability test dominates the equivalent RMZL test

Summary and conclusions

- **Motivation**
  - To improve on current state-of-the-art in terms of techniques that enable the efficient use of processing capacity in hard real-time systems based on multiprocessors.
  - Aimed to improve upon the effectiveness of global FP scheduling without introducing significant additional overheads (e.g. large numbers of context switches)
  - Therefore investigated a minimally dynamic priority algorithm FPZL

Summary and conclusions

- **Contribution**
  - Introduced polynomial and pseudo-polynomial time schedulability tests (Deadline Analysis and Response Time Analysis) for FPZL
  - Improved these tests via calculation of the maximum execution time in the zero-laxity state
  - Test dominate the equivalent tests for global FP
  - Empirical results show that FPZL schedulability tests make a useful improvement on those for global FP particularly in the implicit deadline case
  - Simulation results show that FPZL (and EDZL) are highly effective – still a large gap between simulation and schedulability analysis potentially due to pessimism in the analysis