## Exercise 1 Partial Differentiation

* 1. Given that $z = \exp(xy^2 + y^3x^2)$, compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

* 2. Given that $z = w - y\cos(wx) + x^2$, compute $\frac{\partial z}{\partial w}$, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

** 3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $xz = \ln(y + z)$.

** 4. Schrödingers equation for a single particle moving in free space in two dimensions is given by

$$j\frac{\partial A}{\partial t} = -\frac{\hbar}{2m}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right)$$

where $A(x, y, t)$ is the wavefunction ($j = \sqrt{-1}$). Show that $A(x, y, t) = \exp[j(k_x x + k_y y - \omega t)]$ is a possible solution of this equation, and give the relationship between $\omega$,$k_x$ and $k_y$ for it to be a solution.

** 5. Find the stationary points of $f(x, y) = x\exp(-x^2 - y^2)$ and charaterise them.

## Exercise 2 Optimisation

* 1. Find a gradient descent scheme for minimising $f(x) = \exp\cos x$.
   a) If $x_0 = \pi/2$ then what is the maximum gradient multiplier $a$ which will allow convergence to the closest minimum?

** 2. Find the stationary points of the function

$$f(x, y) = \frac{3}{5}x^3 + \frac{9}{10}x^2y + \frac{6}{5}xy^2 + \frac{8}{15}y^3 + 6x^2 + 9x - \frac{5}{2}y$$

   a) Find the Newton-Raphson update for this function at the point $x = -1, y = -3$.
   b) Characterise the nature of the stationary point at $(-1, 2)$.

*** 3. The minimum distance between a plane and a point in space $\mathbf{x}$ is given by $d = \mathbf{n}.(\mathbf{x} - \mathbf{p})$ where $\mathbf{n}$ is the normal to the plane, and $\mathbf{p}$ is any point on the plane. Using an gradient descent scheme, find the minimum distance between the unit sphere and the plane $x + 2y + 2z = 4$. (Hint: it may be useful to consider spherical polar coordinates for the point.)