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Experimental Signature of Registration Noise in Pulsed Terahertz Systems

Netta Cohen, J. W. Handley, R. D. Boyle
School of Computing, University of Leeds, Leeds, LS2 9JT, UK

Samuel L. Braunstein
Computer Science, University of York, York YO10 5DD, UK

Elizabeth Berry
Academic Unit of Medical Physics, Centre of Medical Imaging Research, University of Leeds, Wellcome Wing, Leeds General Infirmary, Leeds LS1 3EX, UK

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This letter reports results from time domain measurements in a terahertz pulsed imaging system to demonstrate that a mechanical resetting mechanism in the pump-probe delay stage results in a small but resolvable noise signal. In the setup described here, this effect dominates all other sources of noise such as the background Johnson noise or shot noise, and can hence be isolated and analysed in detail. An analysis of the noise signal is used to estimate the physical limitations of the pump-probe system being employed. A comparison of the results with an analytic prediction allows us to formulate a useful and general signature of registration jitter, that should make it easy to detect in any sufficiently narrowband signal.

Keywords: Terahertz pulsed imaging; T-rays ; noise signature; registration jitter.

1. Introduction

This Letter presents measurements and error analysis of a pulsed terahertz (THz) system. THz time domain measurements were performed to detect systematic noise and physical limitations of the THz system. We show that the limiting error in our measurements results from micrometre-scale offsets in an automated resetting mechanism between recordings, leading to a registration error of $O(1\,\text{fsec})$ in the sampling times. While this temporal offset is much smaller than the typical THz wavelength, and significantly smaller than the sampling interval in our setup, the resulting noise signal is nonetheless measurable and appears to dominate over background Johnson noise, shot noise, etc., when these noise sources have been minimised by the lock-in amplifier. Unlike the latter sources of noise, that typically give rise to Gaussian mixture (GMM) models of noise, registration jitter exhibits a very different noise signature and should be treated separately.

In the previous paper [1], we have shown that these GMM models do not apply very well to our THZ time domain measurements. Rather, the noise is modelled by more general stable distributions. In this Letter we analyse the source of the noise in detail and show that by and large it can be attributed to jitter in the registration of the sampling mechanism. We provide analytic confirmation of the results and use these in order to derive the parameters of the system and its physical limitations.

The remainder of the paper is structured as follows. Section 2 describes the experimental system, with emphasis on the pump-probe setup, registration and sampling mechanism. Experimental evidence of registration noise in the time domain measurements is presented in Section 3.1. These results are followed by a general analytic derivation of a matching noise signature for narrowband signals (Section 3.2) and its applicability to THz signals. Further intuition about registration jitter and its distinctive signature is given in the Conclusions. In addition, we demonstrate how detecting registration jitter can teach us about the system's physical limitations.

2. Methods

The experimental system and imaging setup are described in detail in the previous paper [1] (Leeds setup) and further details are available in Refs. [2, 3, 4, 5]. This Letter focuses on time domain measurements acquired under normal atmospheric conditions, involving only the THz generation and detection systems.

In what follows we describe the optical setup used to capture the THz signal. An 80 MHz repetition rate Ti:Sapphire laser (Tsunami, SpectraPhysics) shining on a ZnTe crystal is used to generate the THz source field. The terahertz pulse is focused on an electro-optical detection crystal. The incident terahertz field creates an instantaneous birefringence in the crystal, which is measured with a near-infrared beam. This probe beam is circularly polarised with a quarter waveplate, before the birefringence modulates how elliptical this polarisation is. The difference between the vertical and horizontal components are measured using a Wollaston polarisation splitting prism, and two balanced photodiodes. This balanced detector employs common mode noise rejection by subtracting the 'vertical' signal from the 'horizontal' signal. The difference on the balanced detector is directly proportional to the terahertz radiation incident on the detector crystal.

In order to obtain the electric field of the entire pulse, an optical delay stage (Newport, M-UTM150CC1HL) is used, which lengthens or shortens the path of the probe beam compared to the path of the THz pulses. The location of the delay stage varies the time point at which the snapshot of the terahertz pulse is taken, and enables an entire time series to be collected. The delay stage must obviously be positioned in such a way that a meaningful window into the data is achieved, and this position is known as the initial displacement – a value which is essentially arbitrary to a given system. In addition, a lock-in amplifier [7265 DSP, EG&G/Perkin Elmer (now Signal Recovery)], operated with a large lock-in time constant (100 msec), helps reduce noise levels. Note that such a long time constant effectively eliminates any Johnson noise from the signal.

Of particular importance to us is the resetting mechanism used between runs. At the start of every new pulse acquisition, the delay stage is reset to its initial 'offset' position.

3. Results

3.1. *Time Domain Measurements*

A set of 16 pulses of THz radiation were recorded within a short space of time and with identical acquisition parameters. In each case 256 time points were recorded at 80 fsec intervals, with a time constant of 100 msec and an initial displacement of 22.5 mm. There was a very small deviation between each of the pulses, as demonstrated in Figure 1 which shows all 16 pulses superimposed on the same scale.

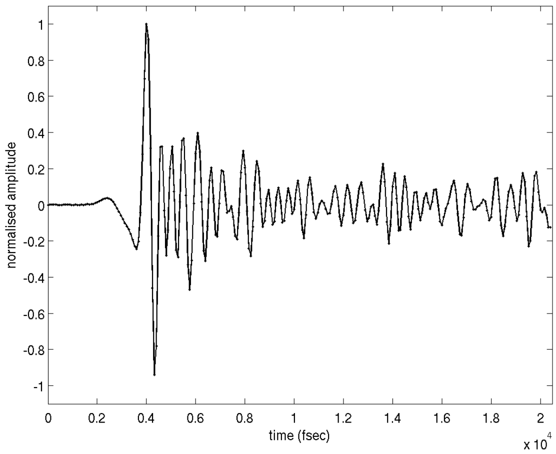

Fig 1. Time domain trace of 16 THz pulses (arbitrary units) acquired under normal atmospheric conditions. The traces exhibit strongly damped oscillation with a centre frequency of about 2 THz. Note that all 16 traces look virtually indistinguishable.

For convenience, we define a mean signal as the average over the 16 pulses. In fact, the noise trace $\mathscr{N}^{(i)}$, defined here as the difference between a particular pulse i and the mean pulse, is not at all independent from the signal. Figure 2 demonstrates that, even though the noise amplitude is very small [Figure 2(a)], the waveform of noise trace shadows the signal trace with a slight delay [Figure 2(b)]. This result, shown here for the first of 16 noise traces, holds equally for all 16 traces. A visual inspection of the delay in the noise shows that it is approximately 140 -160 fsec. A closer examination (seeking the maximal correlation coefficient between the time-delayed signal and noise trace for a range of time delays) gives similar estimates for all 16 pulses. This delay is remarkably close to a quarter cycle of the original signal.

Interestingly, among the 16 pulses recorded, the first few (pulses 1-6) exhibited noise traces as shown in Figure 2(a), whereas the last few (pulses 11-16) appear as near reflections of the former. This effect is demonstrated in Figure 2(c), which shows 12 of the 16 noise traces superimposed after normalisation and up to a sign change of pulses 11-16. The remaining 4 pulses (7-10) had a significantly lower noise amplitude (rms of the difference between the noise trace and the mean signal trace); hence this effect of shadowing the mean signal, while discernible, was somewhat masked by other sources of background noise.

Once the noise signature (140 -160 fsec delay, and potential sign flip) is identified, it is possible to quantify the extent of the correlation between the noise and expected (or mean)

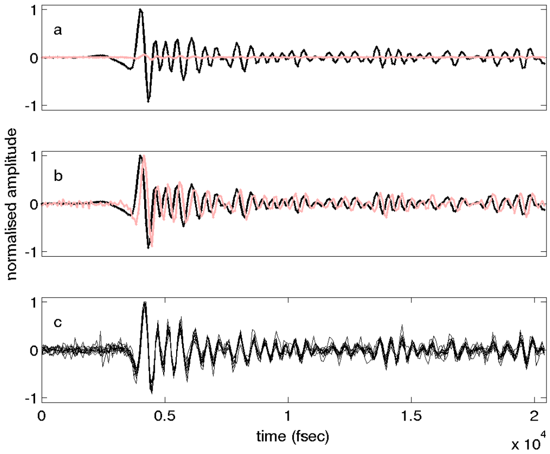

Fig 2. Time traces of the mean signal (average over all 16 pulses) and noise traces $\mathcal{N}^{(i)}$. (a) The mean signal (black) is normalised, and the noise extracted from the first pulse $\mathcal{N}^{(1)}$ is plotted on the same scale (grey). (b) Same as above, but now the noise amplitude is also normalised. The noise signal is delayed by a quarter cycle (just under 2 sample points) relative to the mean signal. (c) A superposition of the time domain of 12 of 16 noise traces, up to amplitude sign flips: Pulses 1-6 plotted upright, and pulses 11-16 are plotted with reflected amplitudes. Pulses 7-10 are omitted (see text for detail).

signal. Figure 3(a) shows the correlation coefficient obtained between the time delayed mean signal and each of the 16 pulses, for optimal delays. Fourteen of 16 noise traces were found to be correlated with the mean signal (with $|C| > 0.6$); 10 of 16 were highly correlated (with $|C| > 0.8$). The high correlation is demonstrated for $\mathcal{N}^{(1)}$ in Figure 3(b).

Note that the correlation coefficients shown in Figure 3(a) reflect a systematic trend from correlated to anti-correlated noise relative to the mean signal. This result suggests that, rather than registration jitter, there may be some systematic drift in the system. We shall return to this point in the next section.

3.2. *Theoretical confirmation*

Consider a signal $f(t)$ that is sampled at times $\bar{\tau}_j = j\Delta$ (i.e., separated by fixed intervals Δ) to yield a time series $f(\bar{\tau}_j)$. For a THz system with a well defined centre frequency ω_0 and bandwidth B, it is convenient to work in the frequency domain:

$$f(\bar{\tau}_j) = \int_B \frac{d\omega}{2\pi} e^{i(\omega+\omega_0)\, j\Delta}\, \tilde{f}(\omega - \omega_0)\,. \tag{1}$$

Now consider the effect of registration jitter: If the same ideal signal is resampled multiple times, then each recording i, sampled at times $\tau_j^{(i)} = j\Delta + \theta_i$, yields a different time series $f(\tau_j^{(i)})$. We assume there is no sampling jitter, so the intervals Δ remain fixed, and the different sets of sampling times in any two recordings only differ by a fixed registration offset θ_i relative to our reference train $\bar{\tau}_j$:

$$f(\tau_j^{(i)}) = \int_B \frac{d\omega}{2\pi} e^{i(\omega+\omega_0)(j\Delta+\theta_i)}\, \tilde{f}(\omega - \omega_0)\,. \tag{2}$$

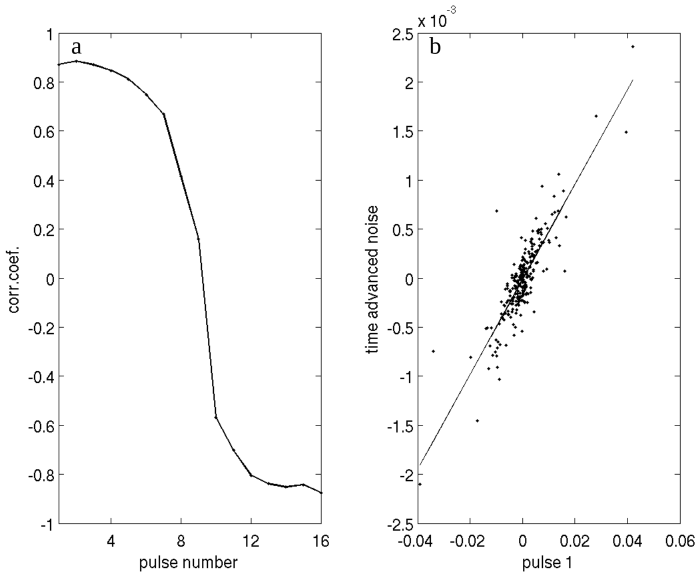

Fig 3. (a) The optimal correlation coefficient between the time-delayed mean signal and each of the 16 noise signals $\mathscr{N}^{(i)}$. The time delays are found for each pulse by seeking the delay that yields the maximum correlation. Optimal time delays all fall in the range 140 -160 fsec. (b) A scatter plot of the noise trace from the first pulse $\mathscr{N}^{(1)}$ versus the mean signal, time delayed by 146 fsec. A linear fit (solid line) is superimposed.

The ideal noise signal $\mathscr{N}^{(i)}(\bar{\tau}_j) = f(\tau_j^{(i)}) - f(\bar{\tau}_j)$ is then given by the difference between equations (1) and (2).

$$\mathscr{N}^{(i)}(\bar{\tau}_j) = \int_B \frac{d\omega}{2\pi}\, e^{i(\omega+\omega_0)\, j\Delta}\, 2i e^{i(\omega+\omega_0)\theta_i/2} \sin\left[\frac{(\omega+\omega_0)\theta_i}{2}\right] \tilde{f}(\omega-\omega_0)\,. \qquad (3)$$

For simplicity, let us assume that the signal is relatively narrowband $B \lesssim \omega_0$. Taking constants out from under the integral and replacing i by a $\pi/2$ phase term then gives

$$\begin{aligned} \mathscr{N}^{(i)}(\bar{\tau}_j) &\simeq -2\sin\left(\frac{\omega_0\theta_i}{2}\right) e^{i\,\omega_0\theta_i/2} \int_B \frac{d\omega}{2\pi} e^{i[(\omega+\omega_0)\, j\Delta - \pi/2]}\, \tilde{f}(\omega-\omega_0) \qquad (4)\\ &\simeq -2\sin\left(\frac{\omega_0\theta_i}{2}\right) e^{i\,\omega_0\theta_i/2}\, f\left(\bar{\tau}_j - \frac{\pi}{2\omega_0}\right) + O\left(\frac{B}{2\omega_0}\right)\,. \end{aligned}$$

Equation (4) tells us that a registration offset θ_i will result in a noise signal that is delayed by a quarter cycle and reversed (i.e., negative) relative to the ideal or expected signal. (An extra sign flip is obtained if θ_i is itself negative, but the positive delay remains unchanged.) Detuning results in higher-order corrections of the order of the relative bandwidth. In fact, THz radiation is typically considered to be 'broadband'. Indeed, the signals treated here have a relatively broad power spectrum, with $|\omega-\omega_0|/\omega_0 \leq 0.36$ spanning 76% of the power. These higher-order corrections degrade our results somewhat but do not appear crucial (see for instance the high correlation coefficients in Fig. 3).

To estimate the registration offset, we could perform a least-squares fit over a specific pulse relative to the mean signal. An explicit formula for this estimate is given by

$$-2\sin\left(\frac{\omega_0\,\theta_i}{2}\right) = \arg\min_c \sum_j \left[c\, f\left(\overline{\tau}^{(i)} - \frac{\pi}{2\omega_0}\right) - \mathscr{N}^{(i)}(\overline{\tau}_j)\right]^2 \approx -\omega_0\theta_i\,, \qquad (5)$$

where c is the estimated noise amplitude for a quarter-wave delay (that may be determined separately). Note that, for small offsets, the amplitude of the noise signal c reduces to $-\omega_0\theta_i$.

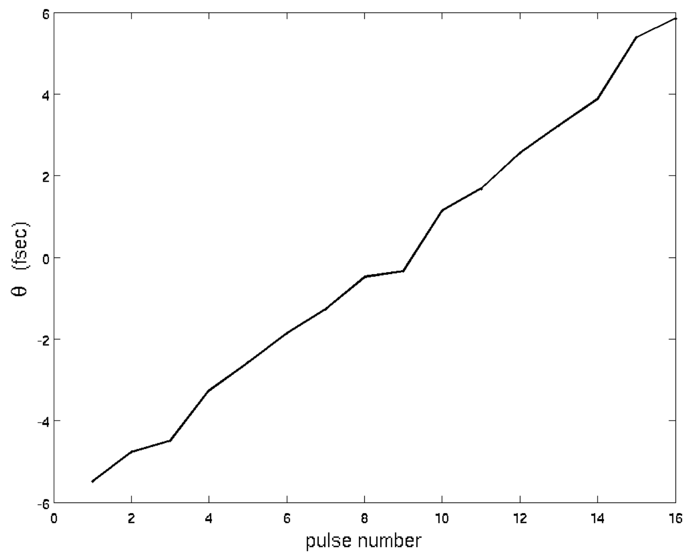

Fig 4. The estimated registration drift from pulse to pulse in fsec, relative to the mean pulse. Note the monotonic, nearly linear drift. spanning a total of approximately 11 fsec, or under 1 fsec per resetting event.

Now, we use Eq. 5 to estimate the registration offsets for the available 16 pulse data. For simplicity, we consider a delay of 2 sample points (160 fsec) as the quarter-wave delay for all pulses. Figure 4 shows this estimated registration offset relative to the mean signal, for each of the pulses. The first striking result is the monotonic increase in the offset from pulse to pulse. Thus, by recognising the signature of registration errors, it is possible to pinpoint a systematic drift in the system that may then be remedied. In fact, according to this estimate, the average registration offset per resetting event is under 1 fsec. Translating this error to an actual resetting distance of the delay stage in the experimental setup yields an average drift of about 0.2 μm per resetting event.

As a final note, the main result (Eq. 4) can straightforwardly be generalised from registration jitter (or drift) to sampling jitter (or drift), where Δ itself fluctuates from one sample point to the next. At any point in time $\tau_j^{(i)}$, we may use the previous point $\tau_{j-1}^{(i)}$ (or even better $\overline{\tau}_{j-1}$ when available) as reference, and then the result in Eq. 4 holds: The magnitude of the jitter determines the amplitude and sign of the error, which will then appear after a quarter cycle delay.

4. Conclusions

The calculation above and its application to THz pulse imaging data demonstrate that registration noise has a general and easy to recognise signature. The basic result is highly intuitive. Given any signal, a sufficiently small and fixed offset in the sampling times yields a difference-signal proportional to the derivative of the original trace. For sinusoidal signals, this trivially translates to a quarter-cycle delay and sign flip. In the experimental setup described here, the registration offset is most likely to be due to a mechanical effect. Small (sub-micron) jitter or drift in the initial displacement of the delay stage in the pump-probe detector setup is the most likely source of the observed temporal offsets.

Remarkably, applying this result to our data shows that minute offsets in registration, that are much smaller even than the sampling intervals of the data, can give significant, clearly discernible noise signals. By estimating the offset in each pulse, it is possible to derive estimates for the physical limitations of the imaging system. It is also possible to characterise the time-series of offsets and hence determine whether the noise is due to random jitter or to systematic drift.

We note that while registration offsets offer the most likely explanation of the data, this has not been explicitly proven in this paper. In particular, no direct observations could be made of the delay stage, to confirm and quantify any drift of the delay stage, or the development of such drift over time. Such an investigation would significantly contribute to the proof of registration offsets in the noise observed here. Additionally, it is not clear how general or system-specific such registration offset problems may be in pump-probe based systems. In general, registration offsets and particular mechanisms that give rise to them have been recognised and studied in a variety of setups (in particular in interferometric setups). The observation of a clear signature of registration effects in THz imaging systems suggests that lessons learnt in these other imaging systems (e.g. Ref. [6]) may be useful here as well.

Specifically, in THz imaging systems, we have found that registration offsets of under 1 fsec give rise to a clearly resolved noise signal. This clear signature can be used as a diagnostic tool if a system becomes noisy to determine whether the delay stage is at fault. It could also be of value as a regular quality assurance measure, perhaps with some ‘acceptable’ noise level build in.

Terahertz radiation is typically broadband, raising questions about the validity of the results and calculations presented here. In fact, Equation 4 shows that not only does this signature hold for general narrowband signals but that it is expected to degrade gracefully for broader band signals. Indeed, we have found that even for our relatively broadband THz oscillations, the noise is dominated by a quarter-cycle delayed image of the signal.

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