Fortschr. Phys. **48** (2000) 5–7, 553–557

Entanglement Swapping as Event-Ready Entanglement Preparation

Pieter Kok[1*]) and Samuel L. Braunstein[1,2]

[1] SEECS, University of Wales, Bangor LL57 1UT, UK
[2] Hewlett-Packard Labs, Mail Box M48, Bristol BS34 8QZ, UK

Abstract

In this contribution the preparation of event-ready entanglement in polarised photons by means of entanglement swapping is studied. This protocol makes use of Gaussian photon-sources. Since the states resulting from this protocol are equivalent to those obtained by mixing two photons in a 50 : 50 beam splitter, the nature of the emerging entanglement is controversial. Several methods to enhance the fidelity of the outgoing state are studied. It is found that at least quantum non-demolition measurements or a quantum computer of some kind is needed to create event-ready entanglement with entanglement swapping.

State preparation is one of the most challenging areas in quantum physics. In particular, we would like to be able to produce entanglement in physical systems such as photons, electrons *et cetera*. Recently, major progress has been made in the preparation of (entangled) photons [1].

Currently, photons are created by letting a high-intensity laser interact with a medium (e.g., a $\chi^{(2)}$-medium) which exhibits a non-linearity of some kind. Examples of this process are squeezing and parametric down-conversion. These photon-sources lie at the heart of experiments like quantum teleportation [2–4] and entanglement swapping [5].

In this contribution we discuss the preparation of so-called *event-ready* entanglement by means of entanglement swapping. A state is said to exhibit event-ready entanglement when it is a *freely propagating* (near) maximally entangled state. The experimental implementation of entanglement swapping was first proposed by Zukowski, Zeilinger, Horne and Ekert [6], and subsequently by Pavičić [7], who explicitly designed his setup to prepare event-ready entanglement. One immediate field of application for event-ready entanglement would be quantum cryptography, quantum computation and information [9].

Here, we are interested in a (near) maximally entangled outgoing state, henceforth called event-ready entanglement. In the case of two linearly polarised photons the maximally entangled states are the Bell states, e.g.,

$$|\Psi^{\pm}\rangle = (|x,y\rangle \pm |y,x\rangle)/\sqrt{2}\,, \tag{1a}$$

$$|\Phi^{\pm}\rangle = (|x,x\rangle \pm |y,y\rangle)/\sqrt{2}\,, \tag{1b}$$

where $|x\rangle$ and $|y\rangle$ are single-photon states with linear polarisations in the x- and y-direction of a given co-ordinate system respectively. Creating two photons which exhibit event-ready entanglement in their polarisation now implies the *outgoing state* (i.e., a physical state, not a wave-function post-selected upon a completely destructive measurement of the outgoing state)

$$\rho_{\rm out} = |\mathrm{Bell}\rangle\,\langle\mathrm{Bell}| + O(\xi) \tag{2}$$

*) E-mail: pieter@sees.bangor.ac.uk

conditioned on detector coincidences, with $|\text{Bell}\rangle$ any of the four polarisation Bell states of Eq. (1) and $\xi \ll 1$. Here we investigate whether entanglement swapping in the proposals mentioned above can be used to obtain event-ready entanglement.

Entanglement swapping is essentially the teleportation of one part of an entangled pair [5, 6, 8]. It is usually described in terms of two (maximally entangled) Bell states:

$$\begin{aligned} |\Psi\rangle_{abcd} &= |\Psi^-\rangle_{ab} \otimes |\Psi^-\rangle_{cd} \\ &= \frac{1}{2} \left(|\Psi^-\rangle_{ad} |\Psi^-\rangle_{bc} + |\Psi^+\rangle_{ad} |\Psi^+\rangle_{bc} \right. \\ &\quad \left. + |\Phi^-\rangle_{ad} |\Phi^-\rangle_{bc} + |\Phi^+\rangle_{ad} |\Phi^+\rangle_{bc} \right). \end{aligned} \tag{3}$$

A Bell detection of modes b and c will select the corresponding Bell state in modes a and d.

The experimental setup used by Pan et al. [5] (as proposed by Zukowski et al. [6] and Pavičić [7]) effectively consists of two type II parametric down-converters which create entangled photon-pairs. In this setup a crystal with a $\chi^{(2)}$ non-linearity is pumped by a high-intensity laser, which we will treat classically (the parametric approximation). The outgoing modes constitute two partially overlapping cones with orthogonal polarisations. In the two spatial modes where the cones intersect we can no longer infer the polarisation of the photons, and as a consequence the two photons become entangled in their polarisation.

In the experiment, one down-converter was pumped twice in opposite direction. This way, a state which is equivalent to a state originating from two independent down-converters was obtained. In order to simplify our discussion, we will treat the experimental setup as if it consists of two down-converters (see Figure 1). One mode of either down-converter is sent into a beam-splitter, the output of which is detected. In the case where both down-converters create a polarisation entangled photon-pair, a coincidence in the photo-detectors D_u and D_v identify the antisymmetric Bell state $|\Psi^-\rangle$ [10].

However, the parametric down-converters do not produce Bell-states [6, 11, 12]. They are part of the class of Gaussian evolutions:

$$|\Psi\rangle = U(t)\,|0\rangle = \exp\left[itH_I/\hbar\right], \tag{4}$$

with

$$H_I = \frac{1}{2} \sum_{ij} \hat{a}_i^\dagger \Lambda_{ij} \hat{a}_j^\dagger + \text{H.c.}, \tag{5}$$

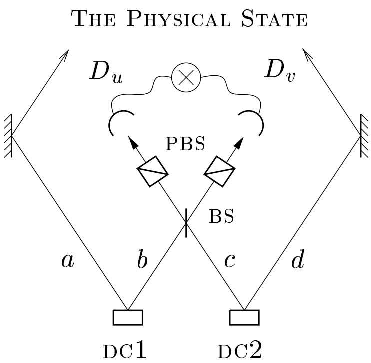

Fig. 1. Entanglement Swapping as Event-Ready Entanglement Preparation, Pieter Kok and Samuel L. Braunstein.
A *schematic* representation of the entanglement swapping setup. Two parametric down-converters (PDC) create states which exhibit polarisation entanglement. One branch of each source is sent into a beam splitter (BS), after which the polarisation beam splitters select particular polarisation settings. A coincidence in detectors D_u and D_v ideally identify the $|\Psi^-\rangle$ Bell state. However, since there is a possibility that one down-converter produces two photon-pairs while the other produces nothing, the detectors D_u and D_v no longer contitute a Bell-detection, and the freely propagating PHYSICAL STATE is no longer a pure Bell state.

where H_I is the interaction Hamiltonian, $\hat{a}_i^\dagger$ a creation operator and Λ_{ij} the components of a (symmetric) matrix. H.c. stands for the Hermitian conjugate. If Λ is diagonal the evolution U corresponds to a collection of single-mode squeezers. In the case of degenerate type II parametric down-conversion used to produce a photon-pair exhibiting polarisation entanglement, the interaction Hamiltonian is

$$H_I = \kappa(\hat{a}_x^\dagger \hat{b}_y^\dagger - \hat{a}_y^\dagger \hat{b}_x^\dagger) + \text{H.c.}\,, \tag{6}$$

with κ a parameter which is determined by the strength of the pump and the coupling of the electro-magnetic field to the crystal.

Up to second order the outgoing state of the down-converter is

$$\begin{aligned}|\Psi\rangle_{ab} &= \left(1-\xi^2\right)|0,0\rangle_{ab} + \xi(|x,y\rangle_{ab} - |y,x\rangle_{ab}) \\ &\quad + \xi^2(|x^2,y^2\rangle_{ab} - |xy,xy\rangle_{ab} + |y^2,x^2\rangle_{ab})\,,\end{aligned} \tag{7}$$

with $\xi \ll 1$, which is a function of κ. Here, $|x^2\rangle$ is a x-polarised mode in a 2 photon Fock state (the case of two y-polarised or an x- and a y-polarised photon are treated similarly). When we are studying coincidences between two down-converters we cannot ignore the fact that with a probability proportional to ξ^2 a single down-converter produces two photon-pairs. After all, the probability of creating one photon-pair in either down-converter is proportional to ξ^2 as well.

Two photons in the outgoing modes of the beam-splitter now no longer necessarily originate from different down-converters, and we can no longer interpret a detector-coincidence at the outgoing modes of the beam-splitter as a projection onto the $|\Psi^-\rangle$ Bell-state. In order to see this, consider a two-photon polarisation state. It is a vector in a Hilbert space generated by (for instance) the basis vectors $|x,x\rangle$, $|x,y\rangle$, $|y,x\rangle$ and $|y,y\rangle$. The $|\Psi^-\rangle$ Bell-state is a superposition of these basis vectors. The key observation is that the two photons described in this Hilbert space occupy *different spatial modes*. In other words, this space should be clearly distinguished from a (truncated) Fock space, where the two photons can occupy the same spatial mode. A detector-coincidence at the outgoing modes of the beam-splitter can thus only be a projection on the $|\Psi^-\rangle$ Bell-state if the incoming modes are both occupied by *one* photon. When two photons no longer necessarily originate from one down-converter there is a possibility that they both enter the beam-splitter at the same input mode, while in the other mode the vacuum state is present. A detector-coincidence at the outgoing modes therefore no longer identifies the $|\Psi^-\rangle$ Bell state.

It turns out that the four outgoing states conditioned on the four different polarisation settings at the output modes of the beam-splitter ((x,x), (x,y), (y,x) and (y,y)) have a remarkably simple form:

$$|\phi_{(x,x)}\rangle_{ad} = |0,y^2\rangle - |y^2,0\rangle \tag{8a}$$

$$|\phi_{(x,y)}\rangle_{ad} = |0,xy\rangle - |y,x\rangle + |x,y\rangle - |xy,0\rangle \tag{8b}$$

$$|\phi_{(y,x)}\rangle_{ad} = |0,xy\rangle + |y,x\rangle - |x,y\rangle - |xy,0\rangle \tag{8c}$$

$$|\phi_{(y,y)}\rangle_{ad} = |0,x^2\rangle - |x^2,0\rangle\,. \tag{8d}$$

These states can also be obtained by sending $|x,x\rangle$, $|x,y\rangle$, $|y,x\rangle$ and $|y,y\rangle$ into a 50:50 beam-splitter respectively. When no distinction between the four possible polarisation settings in the two-fold detector-coincidence is made [5, 6], the state of the two remaining (undetected) modes (the physical state) will be a mixture ρ of the four states in Eqs. (8).

Using a technical mathematical criterion based on the partial transpose of the outgoing state ρ [13], it can be shown that ρ is entangled (ρ satisfies this entanglement criterion). However, it can not be used for event-ready detections of polarisation entanglement since the states in Eqs. (8) are not of the form of Eq. (2).

In Ref. [11], Braunstein and Kimble present possible ways to improve the fidelity of the outgoing state in the teleportation experiment by Bouwmeester et al. [3]. There too, we needed to rule out the possibility that one down-converter creates two photon-pairs, while the other produces nothing. Since the experimental setup considered her closely resembles the setup of the teleportation experiment, one might expect that the remedy given by Braunstein and Kimble will be effective here as well.

However, this is not the case. The first approach was to make sure that in one of the outgoing modes (not involved in Alice's Bell-detection) there was only one photon present. In order to achieve this, a detector cascade was suggested which, upon a two-fold detector coincidence, would reveal a two-photon state. Since in the entanglement swapping experiment there is no *conditional* measurement of the outgoing modes, this approach doesn't work here.

Another way of enhancing the fidelity of the teleported output state in Ref. [3] is to differentiate between the coupling of the two down-converters by lowering the intensity of the pump of one of them. Because of the symmetry of the experimental setup of entanglement swapping, this will amount to outgoing states

$$|\phi_{xx}\rangle \propto \xi_2^2\,|0,y^2\rangle - \xi_1^2\,|y^2,0\rangle \tag{9a}$$

$$|\phi_{xy}\rangle \propto \xi_2^2\,|0,xy\rangle - \xi_1\xi_2(|y,x\rangle - |x,y\rangle) - \xi_1^2\,|xy,0\rangle \tag{9b}$$

$$|\phi_{yx}\rangle \propto \xi_2^2\,|0,xy\rangle + \xi_1\xi_2(|y,x\rangle - |x,y\rangle) - \xi_1^2\,|xy,0\rangle \tag{9c}$$

$$|\phi_{yy}\rangle \propto \xi_2^2\,|0,x^2\rangle - \xi_1^2\,|x^2,0\rangle\,. \tag{9d}$$

Varying ξ_1 and ξ_2 will not allow us to create any of the states which have the form of Eq. (2).

Can we turn any of the states in Eq. (8) into the form of Eq. (2)? Additional photon sources would take us beyond the entanglement swapping protocol and we will not consider them here. Alternatively, we might be able to use a linear interferometer to obtain event-ready entanglement. First, observe that instead of taking Eqs. (8) as the input of the interferometer, we can use $|x,x\rangle$, $|x,y\rangle$, $|y,x\rangle$ or $|y,y\rangle$, since these states are obtained from Eqs. (8) by means of a (unitary) beam-splitter operation. If the linear interferometer described above does not exist for these separable states, neither will it for the outgoing states of the entanglement swapping protocol, since these states only differ by a unitary transformation.

Next, suppose we have a linear interferometer described by the unitary matrix U [14], which transforms the creation operators of the electro-magnetic field according to

$$\hat{a}_i^\dagger \to \sum_j u_{ij}^* \hat{b}_j^\dagger\,, \tag{10}$$

where the u_{ij} are the components of U and i,j enumerate both the modes and polarisations. There is no mixing between the creation and annihilation operators, because photons do not interact with each other. It is easy to show that there are no solutions for u_{ij} when we require

$$\hat{a}_i^\dagger \hat{b}_j^\dagger \to (\hat{c}_x^\dagger \hat{d}_y^\dagger - \hat{c}_y^\dagger \hat{d}_x^\dagger)/\sqrt{2}\,. \tag{11}$$

Substituting Eq. (10) into Eq. (11) generates ten equations for eight variables. Explicit calculation shows that it is not possible to transform any of the states in Eq. (8) into the form of Eq. (2) by means of a linear interferometer alone.

Is there another way to turn any of the states in Eq. (8) into the form of Eq. (2)? Observe that we need to bring a two-photon state to a two-photon Bell state. Ordinary detectors destroy photons, so we need at least sufficiently good *polarisation independent* quantum non-demolition (QND) measurements or a quantum computer of some kind. Furthermore, the correlations (constituting the entanglement) must be preserved. Such QND detectors correspond to technology not yet available for optical photons (for QND detections of microwave photons see Ref. [15]). This means that event-ready detections are not yet possible using entanglement swapping with parametric down-conversion.

In this paper we have studied the possibility of entanglement swapping as a means of the event-ready entanglement preparation of two linearly polarised photons [5–7]. The physical state leaving the entanglement swapping apparatus (i.e., the state which does not need to be detected in order to complete the state preparation) happens to be equivalent to the outgoing state of a 50:50 beam splitter conditioned upon one linearly polarised photon in either input mode. With respect to a related experiment involving quantum teleportation [3], it has been described [11] how to enhance the fidelity of the outgoing state. However, these methods fail here, and we need at least a QND measurement or a quantum computer of some kind to turn the outgoing states of the entanglement swapping experiment into event-ready entanglement.

This research is funded in part by EPSRC grant GR/L91344.

References

[1] P. G. Kwiat, K. Mattle, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **75**, 4337 (1995).
[2] D. Boschi, S. Branca, F. De Martini, L. Hardy and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[3] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger, Nature **390**, 575 (1997).
[4] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble and E. S. Polzik, Science **282**, 706 (1998).
[5] J.-W. Pan, D. Bouwmeester, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **80**, 3891 (1998).
[6] M. Zukowski, A. Zeilinger, M. A. Horne and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[7] M. Pavičić, *Quantum Interferometry*, F. De Martini, G. Denardo and Y. Shih (eds.), VCH Publishing Division I, New York (1996).
[8] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[9] C. H. Bennett and G. Brassard, Proc. IEEE International Conference on Computers, Systems and Signal Processing (New York) 175, IEEE (1984).
[10] S. L. Braunstein and A. Mann, Phys. Rev. A **51**, R1727 (1995).
[11] S. L. Braunstein and H. J. Kimble, Nature **394**, 840 (1998).
[12] P. Kok and S. L. Braunstein, LANL e-print quant-ph/9903074, submitted to Phys. Rev. A.
[13] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996); M. Horodecki, P. Horodecki and R. Horodecki, Phys. Lett. A. **223**, 1 (1996).
[14] M. Reck, A. Zeilinger, H. J. Bernstein and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994).
[15] G. Nogues, A. Rauschenbeutel, S. Osnaghi, M. Brune, J. M. Raimond and S. Haroche, Nature **400**, 239 (1999).