

# Macroscopic quantum superpositions by means of single-atom dispersion

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We show that macroscopic quantum superpositions of the electromagnetic field can be generated through amplification of microscopic quantum superpositions prepared in a single atom. Our scheme has the advantage that dissipation is negligible, and hence the superpositions are not rapidly destroyed.

The principle of the superposition of states is central to quantum mechanics. In some alternative theories, such as the nonlinear generalization of Weinberg,<sup>1</sup> it does not hold. For macroscopically distinguishable states the difficulties associated with quantum-mechanical superpositions are popularly referred to as the Schrödinger's cat problem.<sup>2</sup> Among the more promising suggestions for the experimental investigation of macroscopic quantum superpositions are condensed-matter<sup>3</sup> and optical<sup>4-12</sup> systems.

Research on condensed matter has centered on the possibility of observing coherent oscillation between quantum states of the magnetic flux through superconducting quantum interference devices.<sup>3</sup> An early optical scheme proposed propagating a coherent state through an optical Kerr medium to generate a superposition of two coherent states 180° out of phase.<sup>4</sup> A more recent optical scheme proposes measuring the probe in a quantum nondemolition measuring device to project the signal into a superposition of oppositely phased coherent states.<sup>5,6</sup>

Dissipation rapidly changes macroscopic quantum superpositions into mixtures.<sup>13,14</sup> This has been the major obstacle to their implementation in superconducting quantum interference devices.<sup>3</sup> Various proposed optical systems are also made impractical by field dissipation in the generating medium. Although the micromaser can generate superpositions in the face of cavity losses, the phases of successively injected atoms must be precisely controlled.<sup>11,12</sup> The optical system we propose has negligible dissipation. This is because the medium is a single atom in a vacuum, which eliminates loss mechanisms associated with extended optical media such as Rayleigh scattering in optical fibers. The atom could either be trapped or in an atomic beam. Furthermore, the atom is strongly detuned from resonance with the field, which eliminates atomic spontaneous emission.

Our scheme generates a superposition of macroscopically different coherent states of a single-mode electromagnetic field. The superposed coherent states have the same amplitude but different phases.

Hence the macroscopic separation is in the field mode's phase space rather than in physical space.

It is important to distinguish between the addition of field operators and the superposition of quantum states. The addition of different field modes follows from the linearity of Maxwell's classical electrodynamics and is verified in two-slit interference experiments.<sup>15</sup> However, the superpositions of interest in this Letter are superpositions of quantum states and occur in a single field mode, not between different field modes. The state superposition is specific to standard quantum mechanics.

The basis of our scheme is the dispersive interaction of the field with an atomic transition. The atom is prepared in a superposition of states, and the superposition is transferred to the field. A suitable atomic-state scheme is shown in Fig. 1. For definiteness we discuss the example of a  $J = 1/2$  to  $J = 1/2$  state scheme, although the exact structure is not important. The first step is to prepare a superposition of the atomic states  $|-\rangle_g$  and  $|+\rangle_g$  with a resonant  $\pi/2$  pulse centered at (radio) frequency  $\omega_{RF}$ . Next, a circularly polarized pulse centered at (optical) frequency  $\omega_{OF}$ , strongly detuned from resonance, interacts dispersively with the transition  $|-\rangle_g \leftrightarrow |+\rangle_e$ . Angular momentum conservation prevents the field from driving the  $|+\rangle_g \leftrightarrow |-\rangle_e$  transition. Subsequently, the combined atom-field system is in a superposition state. The atomic states  $|-\rangle_g$  and  $|+\rangle_g$  are correlated with a dis-

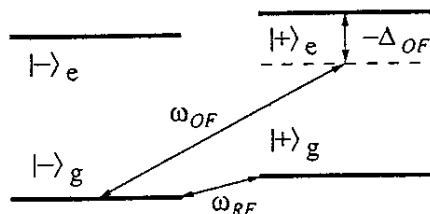


Fig. 1. Atomic-state scheme for the  $J = 1/2$  to  $J = 1/2$  system. A radio-frequency field is used to prepare superpositions of the states  $|-\rangle_g$  and  $|+\rangle_g$ . The optical field interacts dispersively with the transition  $|-\rangle_g \leftrightarrow |+\rangle_e$ .

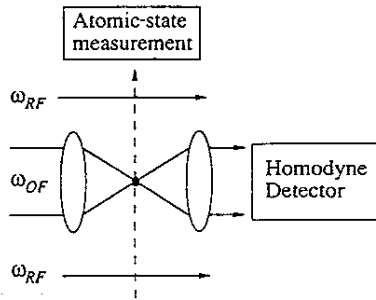


Fig. 2. Schematic of the experimental setup. The dashed line represents an atomic beam, and the solid lines represent the three pulses used in the superposition-generating scheme. The lenses are needed to couple strongly the atom to the optical pulse.

persively phase-shifted field state and an unshifted field state, respectively.

To obtain a purely field superposition the atom must be unitarily transformed by a further  $\pi/2$  radio-frequency pulse. This correlates the field superposition with a superposition of the atomic ground states. Finally, a measurement of the state of the atom projects the field into the coherent-state superposition.

Interaction with more than one atom leads to superpositions of many coherent states with phases ranging between the extreme values. For a macroscopic number of atoms a continuous distribution of phase occurs. The usual refractive index is recovered if no measurement is performed on the atoms.

Dissipation is avoided by performing the experiment in free space rather than in a cavity (Fig. 2). The appropriate field modes are then obtained by decomposing the field into an orthonormal set of time-dependent modes.<sup>16</sup> A simple example of such mode functions are the Nyquist modes.<sup>17</sup> We choose our modes so that one of them is the exciting pulse and the others are left unspecified. They can be ignored provided that large atomic detuning prevents significant scattering into them.

Before discussing our scheme in detail, we review the Jaynes-Cummings interaction of a two-level atom with a single field mode.<sup>18</sup> The mode is represented by a harmonic oscillator having boson annihilation and creation operators  $a$  and  $a^\dagger$ . We denote general upper and lower atomic states by  $|+\rangle$  and  $|-\rangle$ , respectively, and the atomic raising and lowering operators by  $\sigma^+$  and  $\sigma^-$ , respectively. The Hamiltonian for this system is<sup>19</sup>

$$H = \hbar\omega_F a^\dagger a + \hbar\omega_A (\sigma^+ \sigma^- - 1/2) + i\hbar g (a\sigma^+ - a^\dagger\sigma^-),$$

where  $\omega_F$  is the frequency of the field mode and  $\omega_A$  is the frequency of the atomic transition. The coupling strength between the field and the atom,  $g$ , depends on the pulse mode function and the atomic transition; it is assumed to be time independent. The rotating wave approximation implicit in the Hamiltonian is valid for interaction times much longer than  $\omega_F^{-1}$ . The eigenstates of this Hamiltonian are the dressed atomic states ( $|0\rangle|-\rangle$ ,  $|n+\rangle$ ,  $|n-\rangle$ ;  $n = 1, 2, \dots$ ),

$$|n+\rangle = \cos(\phi_n)|n-1\rangle|+\rangle + i \sin(\phi_n)|n\rangle|-\rangle,$$

$$|n-\rangle = -\sin(\phi_n)|n-1\rangle|+\rangle + i \cos(\phi_n)|n\rangle|-\rangle,$$

$$\tan(2\phi_n) = 2gn^{1/2}/\Delta, \quad \Delta = \omega_F - \omega_A,$$

where  $|n\rangle$  is the  $n$ -photon number state of the field and  $\phi_n \in [0, \pi/2]$ . The eigenvalues of these dressed states are

$$\begin{aligned} E(n\pm) &= (n - 1/2)\hbar\omega_F \pm \hbar(1/4\Delta^2 + ng^2)^{1/2} \\ &= (n - 1/2)\hbar\omega_F \pm \hbar g_n, \end{aligned}$$

where  $g_n$  is defined by the last equality. The time evolution of an initial state  $|\Psi(t=0)\rangle = |-\rangle|\alpha\rangle$  with the atom in the ground state  $|-\rangle$  and the field in the coherent state  $|\alpha\rangle$  is

$$\begin{aligned} |\Psi(t)\rangle &= \exp\left(-\frac{1}{2}|\alpha|^2\right) \exp\left(\frac{i}{2}\omega_F t\right) \sum_{n=0}^{\infty} (c_-|n\rangle|-\rangle \\ &\quad + c_+|n-1\rangle|+\rangle) \frac{[\alpha \exp(-i\omega_F t)]^n}{\sqrt{n!}}, \end{aligned}$$

$$c_- = \sin^2(\phi_n) \exp(-ig_n t) + \cos^2(\phi_n) \exp(ig_n t),$$

$$c_+ = -\sin(2\phi_n) \sin(g_n t). \quad (1)$$

The details of our superposition generating scheme are as follows. For the resonant radio-frequency pulses,  $\Delta = 0$ ,  $\phi_n = \pi/4$ , and the states  $|-\rangle_g$  and  $|+\rangle_g$  can be superposed by interaction with a coherent state in the limit  $|\alpha| \rightarrow \infty$ ,  $gt_1 \rightarrow 0$  with  $|\alpha|gt_1 = \pi/4$ . Then  $(n)^{1/2}gt_1$  is approximately constant over the coherent-state distribution, and Eq. (1) implies that

$$\begin{aligned} |\Psi(t_1)\rangle &= \frac{\exp\left(\frac{i}{2}\omega_{RF}t_1\right)}{\sqrt{2}} \left( |\alpha \exp(-i\omega_{RF}t_1)\rangle_{RF} |-\rangle_g \right. \\ &\quad \left. - \left\{ \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{[\alpha \exp(-i\omega_{RF}t_1)]^{n+1}}{[(n+1)!]^{1/2}} |n\rangle_{RF} \right\} |+\rangle_g \right), \end{aligned}$$

where the subscript RF denotes the radio-frequency pulse mode. The field state correlated with  $|-\rangle_g$  is the coherent state of amplitude  $\alpha \exp(-i\omega_{RF}t_1)$ . For large  $|\alpha|$  the field state correlated with  $|+\rangle_g$  is the same coherent state times a phase factor. The atom is then in a suitable atomic superposition state (in this and equations below we have suppressed phase-factors multiplying states):

$$|\Psi(t_1)\rangle = |\alpha \exp(-i\omega_{RF}t_1)\rangle_{RF} \frac{1}{\sqrt{2}} (|-\rangle_g - |+\rangle_g),$$

$$|\alpha| \gg 1.$$

The field state is independent of, or disentangled from, the atomic states and can be ignored in the subsequent analysis.

Having prepared the atomic superposition, the next, and crucial, step is to imprint a superposition on the optical field. In the dispersive limit,  $\Delta \gg 2|\alpha|g$ , the time-evolved state Eq. (1) becomes

$$|\Psi(t_2)\rangle = |\alpha \exp[-i(\omega_{OF} - g_{OF}^2/\Delta)t_2]\rangle_{OF} |-\rangle_g,$$

where  $g_{OF}$  is the Jaynes-Cummings coupling strength to the optical pulse and the subscript OF denotes the optical-frequency pulse mode. On interaction with

the optical pulse, the initial superposition of states,  $(1/\sqrt{2})(|-\rangle_g - |+\rangle_g)$ , evolves into the superposition state

$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} [|\alpha \exp(-i\varphi t_2)\rangle_{\text{OF}} |-\rangle_g + |\alpha \exp(-i\omega_{\text{OF}} t_2)\rangle_{\text{OF}} |+\rangle_g],$$

$$\Delta_{\text{OF}} = \omega_{\text{OF}} - \omega_A \gg 2|\alpha|g_{\text{OF}},$$

$$\varphi = \omega_{\text{OF}} - g_{\text{OF}}^2/\Delta_{\text{OF}}. \quad (2)$$

Note that the  $|+\rangle_g$  part evolves freely, as it is not coupled to any other state by the optical pulse, while the interacting part evolves at the frequency  $\varphi$  rather than the free field frequency  $\omega_{\text{OF}}$ . This is the source of our superpositions. They will be macroscopic, provided that the product of the phase-space angular separation and coherent amplitude exceeds the coherent-state amplitude uncertainty.

The evolved state [Eq. (2)] is an entangled superposition of the atomic and field states. A measurement on the field alone would not reveal the superposition, since the required trace over the orthogonal atomic states  $|-\rangle_g$  and  $|+\rangle_g$  would reduce the field state to a mixture.<sup>20</sup> A simple measurement of the atomic state  $|-\rangle_g$  or  $|+\rangle_g$  would project the field onto the correlated coherent state, again destroying the field superposition. Consequently, the states  $|-\rangle_g$  and  $|+\rangle_g$  must be transformed so that each evolves into superpositions of the other. Each coherent state will then be correlated with both atomic states, and an atomic-state measurement will project the field into a superposition.

Specifically, a radio-frequency  $\pi/2$  pulse identical to that used in the preparation of the initial atomic superposition transforms the state of Eq. (2) into the state

$$|\Psi(t_2 + t_1)\rangle = \frac{1}{\sqrt{2}} \left[ |\alpha \exp(-i\varphi t_2)\rangle_{\text{OF}} \frac{1}{\sqrt{2}} (|-\rangle_g - |+\rangle_g) + |\alpha \exp(-i\omega_{\text{OF}} t_2)\rangle_{\text{OF}} \frac{1}{\sqrt{2}} (|-\rangle_g + |+\rangle_g) \right].$$

As above, the radio-frequency field state is unimportant. An atomic-state measurement then projects the field into the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} [\pm |\alpha \exp(-i\varphi t_2)\rangle_{\text{OF}} + |\alpha \exp(-i\omega_{\text{OF}} t_2)\rangle_{\text{OF}}], \quad (3)$$

where the plus applies if the atomic state was measured to be  $|-\rangle_g$  and the minus applies if state  $|+\rangle_g$  was measured. This coherent-state superposition can be detected by the homodyning technique of Yurke and Stoler.<sup>4</sup> Depending on the local oscillator phase, either the presence of the two distinct coherent states or the fact that they are in a superposition can be verified. To ensure mode matching, the local oscillator must be split off the optical pulse before the interaction. Alternatively, the oscillations in the photon-counting distribution resulting from the superposition Eq. (3) could be detected.<sup>21</sup>

Finally, we discuss the neglect of atomic spontaneous emission in the treatment above. First, it was ignored for the resonant processes of atomic superpo-

sition generation. These superpositions are maintained for times much shorter than the upper-level lifetime; hence the experiment must be performed much more quickly than the state  $|+\rangle_g$  decays. For a radio-frequency transition this should not be a problem. Second, the spontaneous emission out of state  $|+\rangle_g$  was ignored. This is justified provided that the detuning  $\Delta_{\text{RF}}$  is sufficiently large so that the probability for a photon to be spontaneously emitted into the pulse mode is negligible.

In summary, we have presented a scheme for the generation of a quantum-mechanical superposition of macroscopic coherent states of the electromagnetic field. Although we have used the example of a  $J = 1/2$  to  $J = 1/2$  atomic system, the method is readily generalized. In future research we plan to relax the assumptions in our present treatment and to examine the practical aspects of the scheme.

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