Registering incomplete radar images using the EM algorithm

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Abstract

This paper describes an application of the EM (expectation and maximisation) algorithm to the registration of incomplete millimetric radar images. The data used in this study consists of a series of non-overlapping radar sweeps. Our registration process aims to recover transformation parameters between the radar-data and a digital map. The tokens used in the matching process are fragmented line-segments extracted from the radar images which predominantly correspond to hedge-rows in the cartographic data. The EM technique models data uncertainty using Gaussian mixtures defined over the positions and orientations of the lines. The resulting weighted least-squares parameter estimation problem is solved using the Levenberg–Marquardt method. A sensitivity analysis reveals that the data-likelihood function is unimodal in the translation and scale parameters. In fact, the algorithm is only potentially sensitive to the choice of initial rotation parameter; this is attributable to local sub-optima in the log-likelihood function associated with \( \pi/2 \) orientation ambiguities in the map. By adopting Levenberg–Marquardt optimisation we reduce the local convergence difficulties posed by these local rotation maxima. The method is also demonstrated to be relatively insensitive to random measurement errors on the line-segments. © 1997 Elsevier Science B.V.

Keywords: EM algorithm; Incomplete radar images; Non-overlapping radar sweeps; Fragmental line-segments

1. Introduction

Radar imagery is an important means of sensing and mapping elevated features in the landscape. Unfortunately, one of the obstacles to the automatic interpretation of this imagery is the presence of speckle noise [2,32,34]. Rather than being additive in nature, the noise process is multiplicative with the underlying image structure. In fact, the speckle process is most intrusive when the scene contains a physical structure whose spatial scale is of the same order as the wavelength of the radar [2]. Under these conditions speckle produces a significant population of clutter which can confound effective feature extraction and subsequent matching.

In a recent study, we demonstrated how relaxation techniques could be successfully deployed to control clutter in the segmentation and matching of features in conventional synthetic aperture radar images [10,30,31]. The work reported in this paper describes a more ambitious programme of research aimed at matching millimetric Doppler beam sharpened images (DBS) against a digital map. These images differ from their SAR counterparts in a number of important respects. Firstly, the frequency of the radar is of the order of 100 GHz rather than the 10 GHz which is typical of SAR. This means that structures whose size is of the order of a few millimetres appear rough to the radar. The shorter wavelengths employed in the DBS imagery are a consequence of physical constraints imposed upon the dimensions of the resonating cavities in airborne military radars. The second difficulty stems from the imaging geometry. Since the radar is used to sense objects in the line of flight from a low flying aircraft, the images are subject to small angle systematics. Finally, the focusing of the radar means that the scenes are imaged as a series of non-overlapping sweeps interspersed with dead-regions.

The practical goal of the work described in this paper is to develop a navigation aid for airborne vehicles. From a pragmatic standpoint, the aim is to provide an auxiliary radar image guidance system for use in situations when satellite global positioning is unavailable for tactical reasons. In particular, the scenes under study are of rural areas where the principal man-made features available for cartographic matching are linear hedge structures. These are elevated and produce ridge artifacts in the radar images. Since these features are vegetative, they appear rough to the radar and result in specularities. The main obstacle to cartographic matching resides in the fact that because the scene is sensed as a series of non-overlapping sweeps the radar fragments the target hedge structures. This problem is illustrated in the example image shown in Fig. 1. Because the proposed
Recently, the methodological basis of the EM algorithm has attracted renewed interest in the domain of artificial neural networks where it has not only been shown to have an intimate relationship with mean-field annealing [3], but also to provide a convenient framework for hierarchical data processing [17]. Yuille et al. [33] have established the relationship between the EM algorithm and the elastic net of Durbin and Willshaw [8,9]. Novovicova et al. have utilised the algorithm for feature selection; rather than applying the EM algorithm to the expected likelihood, they use as their utility measure the Kullback J-divergence [23].

In the domain of machine vision, the EM algorithm has been exploited for estimating multiple motion parameters [20,1] and for face recognition [21]. Of particular relevance to the work reported in this paper, is the fact that the algorithm has been successfully utilised in the recognition of occluded objects [29] and in the extraction of 3D object pose [15] from relatively uncluttered 2D images. Hornegger and Nieman [15] realise intensity-based object pose estimation under affine transformation using a direct implementation of the classical EM algorithm as originally formulated by Dempster et al. [7]. The method uses basic gradient ascent methods to extract the affine parameters. Viola and Wells [28], on the other hand, have reported a registration method which has many features that are reminiscent of the EM algorithm. The method aims to align raw intensity images so as to maximise a mutual information measure. The maximisation process is realised using stochastic optimisation. Rather than being based on Gaussian mixtures, the mutual information is defined over a set of Parzen estimates. Finally, several authors have considered how the problem of point pattern matching can be addressed using the EM framework. Utans recovers translation parameters [27], while Gold et al. are more ambitious in matching under affine transformation [11].

The registration process realised in this paper is somewhat different from the implementations described above. Firstly, we are concerned with matching line-segments rather than intensity features or dot patterns. Each line is characterised by both position and orientation. Since the radar images are not subject to imaging shear, we confine our attention to the recovery of Euclidean transformation (translation, rotation and scale). The second novel feature resides in the optimisation method used. Rather than adopting a steepest gradient method, we extract the Euclidean parameters using Levenberg–Marquardt optimisation [25]. This is a continuous optimisation method that offers a powerful compromise between steepest gradient and inverse Hessian optimisation; if controlled effectively the method has improved global convergence capabilities.

The outline of the paper is as follows. Section 2 describes the EM registration algorithm. Section 3 describes our radar data and provides experimental details. Finally, Section 4 offers some conclusions and suggests directions for further investigation.
2. The EM algorithm

In this section we detail our representation of the matching process and describe how the underlying set of transformation parameters can be recovered using the EM algorithm. A glossary of the symbols used in the formal development of our algorithm is given in Table 1. The EM algorithm was first introduced by Dempster et al. as a means of fitting incomplete data [7]. The algorithm has two stages. The expectation step involves estimating a mixture distribution using current parameter values. The maximisation step involves computing new parameter values that optimise the expected value of the weighted data likelihood. This two-stage process is iterated to convergence. Although the EM algorithm has been exploited in the recovery of object pose by both Wells [29] and by Hornegger and Nieman [15], the main contribution of this paper is to demonstrate the effectiveness of the algorithm in matching the highly cluttered and incomplete imagery delivered by millimetric radar.

2.1. Representation

Our basic aim is to recover the parameters of the coordinate transformation between the incomplete radar data and a digital map. The tokens used in the matching process are line-segments which are characterised by their mid-point co-ordinates \((x_i,y_i)\) in the image plane and their line-orientation in the image co-ordinate system \(\theta_i\). Each line in the radar-image is represented by a vector \(w_j = (x_i,y_i,\theta_i)^\top\) where \(i\) is the segment index. The available line-data for the radar image is denoted by, the set \(\mathbf{w} = \{w_i, \forall i \in \mathcal{D}\}\) where \(\mathcal{D}\) is the segment index-set. The lines constituting the cartographic model are similarly represented by the set \(z_j = \{z_j, \forall j \in \mathcal{M}\}\). Here \(\mathcal{M}\) is the index-set for the model lines and the \(z_j\) represent the corresponding measurement-vectors. The aim of our matching algorithm is to iteratively recover a parameter-vector \(\Phi^{(n)} = (\phi_1^{(n)}, \ldots, \phi_4^{(n)})^\top\) which describes the Euclidean transformation that brings the radar lines and map lines into registration with one-another. The four components of the parameter vector are as follows; \(\phi_1^{(n)}\) represents the \(x\)-translation, \(\phi_2^{(n)}\) represents the \(y\)-translation, \(\phi_3^{(n)}\) is the rotation and \(\phi_4^{(n)}\) is the relative scale. The registration process is effected by transforming the measurement vectors representing the model-lines in the map into the co-ordinate system of the radar-image. The transformed version of the measurement vector \(z_j\) is given by

\[
\mathbf{z}_j' = F(z_j, \Phi^{(n)})
\]

where the transformation-function \(F\) is defined as follows

\[
F(z_j, \Phi^{(n)}) = U(\Phi^{(n)})z_j + V\Phi^{(n)}.
\]

The matrix \(U(\Phi^{(n)})\) models the scaling and rotation of co-ordinates

\[
U(\Phi^{(n)}) = \begin{pmatrix}
\phi_4^{(n)} \cos \phi_3^{(n)} & -\phi_4^{(n)} \sin \phi_3^{(n)} & 0 \\
\phi_4^{(n)} \sin \phi_3^{(n)} & \phi_4^{(n)} \cos \phi_3^{(n)} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

while the matrix \(V\) selects the translation components for the

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w)</td>
<td>Set of measurements on data-lines</td>
</tr>
<tr>
<td>(\mathcal{D})</td>
<td>Index-set for the data-lines</td>
</tr>
<tr>
<td>(w_j)</td>
<td>Measurement for the data-line indexed (j)</td>
</tr>
<tr>
<td>(\mathcal{M})</td>
<td>Set of measurements on model-lines</td>
</tr>
<tr>
<td>(l_j)</td>
<td>Index-set for the model-lines</td>
</tr>
<tr>
<td>(l_j^{(n)})</td>
<td>Parameter vector at iteration (n) of the algorithm</td>
</tr>
<tr>
<td>(\phi_1^{(n)})</td>
<td>Model translation in the (x)-direction</td>
</tr>
<tr>
<td>(\phi_2^{(n)})</td>
<td>Model translation in the (y)-direction</td>
</tr>
<tr>
<td>(\phi_3^{(n)})</td>
<td>Model rotation parameter</td>
</tr>
<tr>
<td>(\phi_4^{(n)})</td>
<td>Model scaling parameter</td>
</tr>
<tr>
<td>(F(z_j, \Phi^{(n)}))</td>
<td>Euclidean transformation of model-measurement (z_j) under the parameter-vector (\Phi^{(n)})</td>
</tr>
<tr>
<td>(U(\Phi^{(n)}))</td>
<td>Scale-rotation component of the Euclidean transformation</td>
</tr>
<tr>
<td>(V)</td>
<td>Translation component of the Euclidean transformation</td>
</tr>
<tr>
<td>(p(\mathbf{w}</td>
<td>\Phi^{(n)}))</td>
</tr>
<tr>
<td>(p(\mathbf{w}_j</td>
<td>z_j, \Phi^{(n)}))</td>
</tr>
<tr>
<td>(P(z_j</td>
<td>\mathbf{w}_j, \Phi^{(n)}))</td>
</tr>
<tr>
<td>(\alpha_j^{(n)})</td>
<td>Mixing proportion for the model-line indexed (j) at the iteration (n) expectation step</td>
</tr>
<tr>
<td>(\epsilon_j^{(n)}(\Phi^{(n)}))</td>
<td>Error-residual between data-line (i) and model-line (j) at iteration (n)</td>
</tr>
<tr>
<td>(\Sigma)</td>
<td>Variance–covariance matrix for the error-residuals</td>
</tr>
<tr>
<td>(Q(\Phi^{(n+1)}</td>
<td>\Phi^{(n)}))</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Levenberg–Marquardt control parameter</td>
</tr>
</tbody>
</table>
2.2. Expectation

Basic to our philosophy of exploiting the EM algorithm is the idea that every line-segment in the radar data can in principle associate to each of the lines in the digital map with some a posteriori probability. This modelling ingredient is naturally incorporated into the fitting process by developing a mixture model over the space of potential matching assignments. The expectation step of the EM algorithm provides an iterative framework for computing the a posteriori matching probabilities using Gaussian mixtures defined over a set of transformation parameters.

The EM algorithm commences by considering the conditional likelihood for the data-line measurements \( w_i \) given the current set of transformation parameters, \( \Phi^{(n)} \). The algorithm builds on the assumption that the individual data items are conditionally independent of one-another given the current parameter estimates, i.e.

\[
p(w | \Phi^{(n)}) = \prod_{i \in D} p(w_i | \Phi^{(n)}).
\]

Each of the component densities appearing in the above factorisation is represented by a mixture distribution defined over a set of putative model-data associations between the two sets of line-tokens

\[
p(w_i | \Phi^{(n)}) = \sum_{j \in M} p(w_i | z_j \Phi^{(n)}) P(z_j | \Phi^{(n)}).
\]

The ingredients of the above mixture density are the component conditional measurement densities \( p(w_i | z_j \Phi^{(n)}) \) and the mixing proportions \( P(z_j | \Phi^{(n)}) \). The conditional measurement densities represent the likelihood that the data-line measurement \( w_i \) originates from the model-line indexed \( j \) under the prevailing set of transformation parameters \( \Phi^{(n)} \).

We use the shorthand notation \( \alpha_j^{(n)} = P(z_j | \Phi^{(n)}) \) to denote the mixing proportions. These quantities provide a natural mechanism for assessing the significance of the individual model-lines in explaining the current data-likelihood. For instance, if \( \alpha_j^{(n)} \) approaches zero, then this indicates that there is no matching line in the data. In other words, the mixture model provides a natural way of accommodating missing or occluded model segments. It is important to stress that the mixing proportions are iteration dependent, being conditioned upon the current parameter values.

Conventionally, maximum-likelihood parameters are estimated using the complete log-likelihood for the available data

\[
L(\Phi^{(n)}, w) = \sum_{i \in D} \ln p(w_i | \Phi^{(n)}).
\]

In the case where the conditional measurement densities are univariate Gaussian, then maximising the complete likelihood function corresponds to solving a system of least-squares equations for the transformation parameters. By contrast, the expectation step of the EM algorithm is aimed at estimating the log-likelihood function when the data under consideration is incomplete. In our line-matching example this incompleteness is a consequence of the fact that we do not know how to associate tokens in the in the radar image and their counterparts in the cartographic map. In other words, we need to average the log-likelihood over the space of potential line associations. In fact, it was Dempster et al. [7] who observed that maximising the weighted log-likelihood was equivalent to maximising the conditional expectation of the log-likelihood for a new parameter set given an old parameter set. For our matching problem, maximisation of the expectation of the conditional likelihood, i.e. \( E[L(\Phi^{(n+1)}, w) | \Phi^{(n)}, w] \), is equivalent to maximising the weighted log-likelihood function

\[
Q(\Phi^{(n+1)} | \Phi^{(n)}) = \sum_{i \in D} \sum_{j \in M} P(z_j | w_i, \Phi^{(n)}) \ln p(w_j | z_j, \Phi^{(n+1)}).
\]

The a posteriori probabilities \( P(z_j | w_i, \Phi^{(n)}) \) play the role of matching weights in the expected likelihood. We interpret these weights as representing the probability of match between the data line indexed \( i \) and the model-line indexed \( j \). In other words, they represent model-data affinities. Using the Bayes rule, we can rewrite the a posteriori matching probabilities in terms of the components of the conditional measurement densities appearing in the mixture model in eqn (6)

\[
P(z_j | w_i, \Phi^{(n)}) = \frac{\alpha_j^{(n)} p(w_i | z_j, \Phi^{(n)})}{\sum_{j' \in M} \alpha_{j'}^{(n)} p(w_i | z_j', \Phi^{(n)})}.
\]

The mixing proportions are, computed by averaging the a posteriori probabilities over the set of data-lines, i.e.

\[
\alpha_j^{(n+1)} = \frac{1}{|D|} \sum_{i \in D} P(z_j | w_i, \Phi^{(n)}).
\]

In order to proceed with the development of a line registration process we require a model for the conditional measurement densities, i.e. \( p(w_i | z_j, \Phi^{(n)}) \). Here we assume that the required model can be specified in terms of a multivariate Gaussian distribution. The random variables appearing in these distributions are the error residuals for the position and orientation predictions of the jth model line delivered by the current estimated transformation parameters. Accordingly we write

\[
p(w_i | z_j, \Phi^{(n)}) = \frac{1}{(2\pi)^{3/2} \sqrt{\Sigma}} \exp \left[ -\frac{1}{2} e_{ij}(\Phi^{(n)})^T \Sigma^{-1} e_{ij}(\Phi^{(n)}) \right].
\]
In the above expression \( \Sigma \) is the variance-covariance matrix for the vector of error-residuals \( \epsilon_i(\Phi^{(n)}) = w_i - F(z_i, \Phi^{(n)}) \) between the components of the predicted measurement vectors \( z_i \), and their counterparts in the data, i.e. \( w_i \). Formally, the matrix is related to the expectation of the outer-product of the error-residuals, i.e. \( \Sigma = E(\epsilon_i(\Phi^{(n)})\epsilon_i(\Phi^{(n)^T})) \). Accordingly, we compute the following estimate of \( \Sigma \)

\[
\hat{\Sigma} = \frac{\sum_{i \in D} \sum_{j \in E} P(z_i \mid w_i, \Phi^{(n)}) \epsilon_i(\Phi^{(n)}) \epsilon_i(\Phi^{(n)^T})}{\sum_{i \in D} \sum_{j \in E} P(z_i \mid w_i, \Phi^{(n)})}.
\]  

(12)

With these ingredients, and using the shorthand notation \( q_{ij}^{(n)} = P(z_i \mid w_i, \Phi^{(n)}) \) for the a posteriori matching probabilities, the expectation step of the EM algorithm simply reduces to computing the weighted squared error criterion

\[
Q'(\Phi^{(n+1)} \mid \Phi^{(n)}) = \frac{1}{2} \sum_{i \in D} \sum_{j \in E} q_{ij}^{(n)} \epsilon_i(\Phi^{(n)})^T \hat{\Sigma}^{-1} \epsilon_i(\Phi^{(n)}).
\]  

(13)

In other words, the a posteriori probabilities \( q_{ij}^{(n)} \) effectively regulate the contributions to the likelihood function. Matches for which there is little evidence contribute insignificantly, while those which are in good registration dominate.

2.3. Maximisation

The maximisation step aims to locate the updated parameter-vector \( \Phi^{(n+1)} \) that optimises the quantity \( Q(\Phi^{(n+1)} \mid \Phi^{(n)}) \), i.e.

\[
\Phi^{(n+1)} = \arg \max_{\Phi} Q'(\Phi \mid \Phi^{(n)}).
\]  

(14)

We solve the implied weighted least-squares minimisation problem using the Levenberg–Marquardt technique [25,18,22]. This non-linear optimisation technique offers a compromise between the, steepest gradient and inverse Hessian methods. The former is used when close to the optimum while the latter is used far from it. In other words, when close to the optimum, parameter updating takes place with step-size proportional to the gradient \( \nabla_{\Phi} Q'(\Phi \mid \Phi^{(n)}) \). When far from the optimum the optimisation procedure uses second-order information residing in the Hessian, \( H \), of \( Q'(\Phi \mid \Phi^{(n)}) \); the corresponding step-size for the parameter vector \( \Phi \) is \( H^{-1} \nabla_{\Phi} Q'(\Phi \mid \Phi^{(n)}) \). Central to the Levenberg–Marquardt method is the idea of exerting control over these two update modes using a positive parameter \( \lambda \). This parameter defines the elements of the matrix \( \Lambda \).

\[
\Lambda_{k,i} = \begin{cases} 
1 + \lambda & \text{if } k = i, \\
1 & \text{otherwise.}
\end{cases}
\]  

(15)

According to the Levenberg–Marquardt method the stepsize \( \delta \Phi_i \) for the parameter \( \Phi_i \) is found by solving the following set of linear equations

\[
\sum_{i=1}^{d} \Lambda_{k,i} \frac{\partial^2 Q'(\Phi \mid \Phi^{(n)})}{\partial \Phi_k \partial \Phi_i} \delta \Phi_i = \frac{\partial Q'(\Phi \mid \Phi^{(n)})}{\partial \Phi_k}.
\]  

(16)

The parameter \( \lambda \) is chosen to be large if the log-likelihood increases when an optimisation step is taken; in this case the optimisation process operates in steepest gradient mode. If on the other hand, the expected likelihood decreases then \( \lambda \) is reduced towards zero; in this case the optimisation process operates in inverse Hessian mode. When controlled effectively, this method is less prone to local convergence than the standard steepest gradient descent method, while offering efficiency gains over the inverse Hessian method. In fact, we find that the Levenberg–Marquardt optimisation process converges in 5–10 iterations. Steepest-gradient takes up to ten times longer to converge. Although our research-code is not optimised for efficient execution, typical registration experiments involving up to 100 line tokens take tens of seconds on an SGI R10000 workstation.

2.4. Iteration

To conclude this section we describe how the EM algorithm is iterated according to the following steps

- **Initialisation**: The algorithm commences from an initial estimate of the parameter-vector \( \Phi^{(0)} \). The initial mixing proportions are set uniformly across the set of model lines, i.e. \( P(z_i \mid \Phi^{(0)}) = 1/|M| \). The initial a posteriori probabilities \( q_{ij}^{(0)} \) are estimated using the Gaussian density given in eqn (11) and the Bayes normalisation of eqn (9).

- **Expectation**: The expectation step is iterated by using the current parameter vectors to compute density estimates using eqn (11). The a posteriori probabilities are computed by substituting the computed densities into eqn (10). Updated mixing proportions are estimated by averaging the a posteriori probabilities over the data elements using eqn (10).

- **Maximisation**: Updated parameter which maximise the expected-likelihood function are iteratively computed using the Levenberg–Marquardt method outlined in eqn (16).

The only heuristically controlled parameter is the Levenberg–Marquardt threshold switch \( \lambda \). Drawing on conventional wisdom concerning the control of this optimisation algorithm [25], we have used the value \( \lambda = 10 \) in the majority of our experiments.

2.5. Relationship with robust parameter estimation

In passing, it is worth commenting on the relationship between the EM algorithm and the iterative re-weighting
strategies employed in robust estimation [19,14,26].
Robust-estimation aims to locate parameters which mini-
mise an error criterion of the form

$$K(\Phi^{(n)}) = \sum_{i \in D} \sum_{j \in M} \rho\left(\frac{\eta_{ij}^{(n)}}{s}\right),$$

(17)

where $\rho(\eta_{ij}^{(n)}/s)$ is an error function defined over the set of
residuals $\eta_{ij}^{(n)}$ between the model and the data at iteration $n$ of
the algorithm. The quantity $s$ is a parameter that controls
shape of the error-kernel. The model parameters (i.e. the
components of the Euclidean transformation) are iteratively
computed by solving the set of saddle-point equations

$$\frac{\partial K(\Phi^{(n)})}{\partial \phi_k} = \sum_{i \in D} \sum_{j \in M} \Gamma(\eta_{ij}^{(n)}) \frac{\partial \eta_{ij}^{(n)}}{\partial \phi_k} = 0,$$

(18)

where the so-called influence-function $\Gamma(\eta_{ij}^{(n)})$ is related to
the error-function $\rho(\eta_{ij}^{(n)})$ in the following manner

$$\Gamma(\eta_{ij}^{(n)}) = \frac{\partial \rho(\eta_{ij}^{(n)})}{\partial \eta_{ij}^{(n)}}.$$

(19)

The role of the influence function is to generate iteration-
dependent weights which exclude data-outliers in the robust
estimation of parameters. At iteration $n$ of the algorithm the
weight associated with the residual-error $\eta_{ij}$ is returned by the
function

$$w(\eta_{ij}) = s \Gamma(\frac{\eta_{ij}}{s}).$$

(20)

With the weights defined above, the saddle-point equations
for robust parameter estimation are as follows

$$\sum_{i \in D} \sum_{j \in M} w(\eta_{ij}) \eta_{ij} \frac{\partial \eta_{ij}}{\partial \phi_k} = 0.$$  

(21)

For comparison it is informative to write down the corre-
sponding set of saddle-point equations for the expected
likelihood function $Q(\Phi^{(n-1)} \mid \Phi^{(n)})$, i.e.

$$\frac{\partial Q(\Phi \mid \Phi^{(n)})}{\partial \phi_k} = 0.$$  

(22)

Substituting for the expected likelihood function from eqn
(13) and exploiting the derivative identities for matrix
quadric form

$$\sum_{i \in D} \sum_{j \in M} \eta_{ij} \epsilon_{ij}(\Phi^{(n-1)})^T \Sigma^{-1} \epsilon_{ij}(\Phi^{(n)}) = 0.$$  

(23)

The two sets of saddle-point equations take on a striking
similarity when the covariance matrix $\Sigma = s^2 I$, where $I$ is the
3 $\times$ 3 identity matrix. Under these conditions the Mahalanobis
distance becomes equal to the squared Euclidean error-residual, i.e.

$$\left(\frac{\eta_{ij}^{(n)}}{s}\right)^2 = \epsilon_{ij}(\Phi^{(n)})^T \Sigma^{-1} \epsilon_{ij}(\Phi^{(n)}).$$  

(24)

When the relationship between the residuals and the model-
parameters is linear, then both sets of saddle-point equations
lead to fixed-point iteration schemes for the parameters. In the
case of our Euclidean model, the relationship is non-
linear and the saddle-point equations are intractable
in closed-form. Instead, they are solved using the
Levenberg–Marquardt method described in Section 2.3.

There are a number of fundamental differences between
the EM algorithm and robust-estimation which need to be
underlined in more detail. Stated in terms of the saddle-
point equations it is tempting to draw an analogy between the a posteriori matching probabilities $q_{ij}^{(n)}$ and the robust
weighting function $w(\eta_{ij})$. Our first point of contrast is to
stress that whereas the a posteriori probabilities are normal-
ised over the state-space of matches, there is no corre-
sponding normalisation condition for the robust weights – in other
words the weights are not probabilities. Nonetheless, it is
interesting to pursue the assumption that the covariance
matrix is diagonal and hence elucidate the corresponding
form of the error-kernel for robust estimation. According to
Li [26] the kernel is related the first-moment of the
weighting function in the following manner

$$\rho(\frac{\eta_{ij}}{s}) = 2 \int_{-\infty}^{\infty} \xi w(\xi) d\xi.$$  

(25)

Using the definition for the a posteriori probabilities in eqn
(9) and approximating the application of the Mahalanobis
distance in eqn (24) to the components of the mixture distribution
appearing in eqn (11), we arrive at the following
expression for the robust error-kernel

$$\rho(\frac{\eta_{ij}}{s}) = 2 \int_{-\infty}^{\infty} \xi \alpha_j^{(n)} \exp\left[-\frac{1}{2} \left(\frac{\xi}{s}\right)^2\right] d\xi.$$  

(26)

The main point to be drawn from this expression is that the
shape of the weighting function is determined by the line-
mixing proportions $\alpha_j^{(n)}$ which are themselves iteration
dependant. In other words, the shape of the error-kernels
is not only iteration dependant, it is also specific to the
model-line being matched. To make this point explicit, we
note that the kernel for the $j$th model line at iteration $n$ of
the algorithm is given by

$$\rho_j^{(n)}(\frac{\eta_{ij}}{s}) = 2 \int_{-\infty}^{\infty} \xi \alpha_j^{(n)} \exp\left[-\frac{1}{2} \left(\frac{\xi}{s}\right)^2\right] d\xi.$$  

(27)

Finally it is important to stress that our EM approach
exploits the full covariance structure of the data.
Specifically, the covariance matrix Σ is re-estimated at each iteration. This is to be contrasted, for instance, with Li’s adaptive influence function [26]. Here the parameter s is used as a control variable which is incrementally reduced (or annealed) with successive iterations. However, we acknowledge that there are alternatives, most notably computing s using the median absolute difference estimator [14].

3. Experiments

The overall goal of the study reported in this paper is the registration of partial radar images against a digital map. Fig. 1 shows a typical radar image. The images are of rural areas in which the principal man-made linear-features available for matching are hedge-rows. The radar data is delivered as a series of non-overlapping sweeps interspersed with substantial dead-regions. Within each sweep there is both a significant oriented background texture and a systematic variation in background intensity. The linear hedge structures may extend across several adjacent sweeps and are in consequence likely to be broken or fragmented. In other words, the map registration process must be capable of accommodating the matching of single model lines to multiple data-line fragments. It must also accommodate the possibility of missing matches for model-lines which fall into the dead-regions.

The radar imagery is reconstructed using gyroscopic cockpit navigation data to correct for aircraft motion pitch and yaw. As a result there is anticipated to be little affine shear and a Euclidean imaging model is adequate for effective registration.

3.1. Linear feature extraction

Before we can commence with the registration process, we must extract a set of line-segments for matching. In other words, we must provide a means of characterising the radar reflections from hedge-structures. Notwithstanding the difficulties associated with the partial sweeps, there are a number of obstacles to the characterisation of hedges from their radar reflections in speckle noise. In the first instance, the structures themselves are of variable width. Moreover, there are position and orientation dependant shadow artifacts which introduce an anti-symmetric component into the symmetric intensity profile of idealised hedge reflections. It is for these reasons that the hedge radar reflections are not well characterised by idealised even-symmetry fixed-width line-detectors such as the directional second derivative of Gaussian or the cosine-phase Gabor filter. Instead, we capture the variable width and mixed symmetry structure of the intensity profiles using a bank of sine and cosine phase Gabor filters of multiple orientation and scale [12]. Here we adopt a statistical approach based on principal components analysis to find linear combination of these filter responses for optimal hedge-enhancement. Fig. 2 shows the enhanced hedge-features obtained by applying the filter combinations to the raw radar image.

The next task is the re-enforcement of the filter responses to deliver connected linear-features. This second processing step is performed using a Bayesian relaxation scheme which utilises a statistical model of the of the filter-bank response [13]. The linear filter response combinations are used as input to a relaxation operator which iterates to enhance local connectivity using a dictionary of local line-structures [10]. This process of line-enhancement also draws on a statistical model of the filter-bank noise response [12]. According to this model, the modulus of the combined filter response follows a distribution which is a product of Bessel-function and Rayleigh components. The Rayleigh distribution models the uncorrelated noise component in the image while the Bessel distribution models the correlated noise due to multiplicative speckle. The two parameters of this model are the uncorrelated noise-variance together with a noise-correlation parameter. It is this second parameter that controls the relative importance of the Rayleigh and Bessel components in the filter response distribution. An example of the filter response distribution and the predictions of the noise model are shown in Fig. 3. In this example the uncorrelated and correlated noise variances are in the ratio 1.2:1.

Once stable line features are to hand, candidate line-segments may be extracted. Fig. 4 shows the detected straight-line segments. Although most of the genuine hedge-features are detected, there is a significant population of extraneous lines.

3.2. Matching experiments

The Ordnance Survey map data used in our matching
The line-gradient distribution with the fitted noise model.

Fig. 3.

The line-segments extracted from the radar image.

Fig. 4.

The Ordnance survey map data.

Fig. 5.
Fig. 6. Line segments extracted from the ordnance survey map data.

function of the scale, rotation and translation parameters. Fig. 8 shows the expected log-likelihood as a function of scale and translation, while Fig. 9 is a plot of expected log-likelihood as a function of rotation and scale. It is interesting to note that while the scale-translation behaviour is unimodal, the rotation-scale plot contains local sub-optima. Closer inspection of Fig. 9 reveals that these suboptima are associated with rotations of $\pi/2$. This rotation ambiguity is attributable to the fact that the hedge-rows in the digital map are organised into rectangular field-structures.

To illustrate some of the advantages of using the Levenberg–Marquardt method to extract optimal parameters, we provide some comparison with steepest gradient. Our aim here is to illustrate that the convergence of the steepest gradient method is more susceptible to local rotation maxima. Fig. 10 shows an initial configuration in which the map is rotated at an angle of $20^\circ$ with respect to the image; there are also scale and translation differences. Fig. 11 shows the final matching configuration obtained with the Levenberg–Marquardt method. For comparison, the result obtained with steepest gradient alone are shown

Fig. 7. Successive iterations of the map-fitting process. The algorithm converges after six iterations. The second image in the sequence is equivalent to the result of maximum likelihood estimation.

Fig. 8. The data log-likelihood function as a function of translation and scale (note that the function is unimodal).

Fig. 9. The data log-likelihood function as a function of scale and rotation (note that there are sub-optima associated with rotations of the model by angle multiples of $\pi/2$).
in Fig. 12. The main feature to note is that steepest gradient method fails to locate a consistent match. In other words, the Levenberg–Marquardt method is less sensitive to initial model orientation.

Our final set of experiments aims to evaluate the sensitivity of our method to random perturbations of the line-segments. Here we aim to illustrate the noise sensitivity of the match when the endpoints of the line-segments are subjected to Gaussian position errors of known variance. Fig. 13 shows the fraction of lines correctly matched as a function of the line-end position error expressed as a fraction of the average interline spacing. The three curves correspond to different levels of initial line-displacement from the ground-truth solution. The upper curve corresponds to an average initial displacement which is 10% of the average interline spacing, in the case of the middle curve the displacement is 50% while in the case of the lower curve the displacement is 100%. The main conclusions from this plot is that the registration accuracy falls linearly with random measurement errors and that the method can not tolerate initialisation errors significantly greater than 50%.

4. Conclusion

We have detailed a technique for registering incomplete radar images which uses the EM algorithm to estimate transformation parameters using measurements provided by line-segments. The registration technique is only susceptible to local convergence if the initial rotation parameter is poorly estimated. In fact, the data log-likelihood function is unimodal in scale and translation. We aim to minimise the difficulties associated with local convergence by adopting the Levenberg–Marquardt optimisation method. Because both the model-line mixing proportions and the error-residual variance-covariance matrix are estimated in the expectation step of the algorithm, the only heuristically chosen parameter is the Levenberg–Marquardt switch threshold \( \lambda \) used in the maximisation step. However, the value of this parameter is intrinsic to the optimisation method rather than to the image-data being used.

There are clearly a number of ways in which the work described in this paper can be extended. Although several
authors have reported line registration algorithms [4,22] these are invariably based on essentially ad hoc cost functions. One natural extension of our methodology is the recovery of 3D pose from 2D images [15]. Finally, the EM framework reported in this paper is naturally extensible to multiple model matching [1,20].

5. For further reading

[6,16,24]

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References